

## How Many of Each Kind?

Abby and Bing Woo own a small bakery that specializes in cookies. They make only two kinds of cookies—plain and iced. They need to decide how many dozens of each kind of cookie to make for tomorrow.



The Woos know that each dozen of their plain cookies requires 1 pound of cookie dough (and no icing), and each dozen of their iced cookies requires 0.7 pounds of cookie dough and 0.4 pounds of icing. The Woos also know that each dozen of the plain cookies requires about 0.1 hours of preparation time, and each dozen of the iced cookies requires about 0.15 hours of preparation time. Finally, they know that no matter how many of each kind they make, they will be able to sell them all.

The Woos' decision is limited by three factors.

- The ingredients they have on hand—they have 110 pounds of cookie dough and 32 pounds of icing.
- The amount of oven space available—they have room to bake a total of 140 dozen cookies for tomorrow.
- The amount of preparation time available—together they have 15 hours for cookie preparation.

Why on earth should the Woos care how many cookies of each kind they make? Well, you guessed it! They want to make as much profit as possible. The plain cookies sell for 6.00 a dozen and cost \$4.50 a dozen to make. The iced cookies sell for \$7.00 a dozen and cost \$5.00 a dozen to make.

*How many dozens of each kind of cookie should Abby & and Bing make so that their profit is as high as possible?*

### **Your Assignment**

Imagine that your group is a business consulting team, and the Woos have come to you for help. Of course, you want to give them the right answer. But you also want to explain to them clearly how you know that you have the best possible answer so that they will consult your group in the future.

You may want to review what you already know from earlier work on this problem. Look at your notes and earlier assignments. Then prepare a presentation for the Woos. Your presentation should cover these items,

- An answer to the Woos' dilemma, including a summary of how much cookie dough, icing, and preparation time they will use, and how many dozen cookies they will make altogether
- An explanation for the Woos that will convince them that your answer gives them the most profit
- Any graphs, charts, equations, or diagrams that are needed as part of your explanation

You should prepare your presentation based on the assumption that the Woos do not know the techniques you have learned in this unit about solving this type of problem.

**How Many of Each Kind?**

***So, What Does It All Mean???***



<b><i>Word or Phrase</i></b>	<b><i>Meaning</i></b>

# How Many of Each Kind???

## Linear Programming *Constraints and Objective Function*



Type of Cookie	Amount of Cookies (dozens)	Unit Amount of Dough (lbs per dozen)	Total Dough (lbs)	Unit Amount Icing (lbs per dozen)	Total Icing (lbs)	Unit Prep Time (hrs per dozen)	Total Prep Time (hrs)	Unit Profit (\$ per dozen)	Total Profit (\$)
<b>Limits/ Objectives</b>		-----		-----		-----		-----	

**Constraints:** Amount of Cookies:  
(oven capacity)

**Total Dough:**

**Total Icing:**

**Total Prep Time:**

**Objective Function:**

**Natural Constraints:**

**How Many of Each Kind?**

***Where's the MAXIMUM Profit???***



<b>Corner Point (x, y)</b>	<b>Objective Function Profit (x, y) =</b>	<b>Value of Objective Function at Corner Point</b>

**Your Conclusions:**

**ANSWER KEY: ORIGINAL CLASSWORK PROBLEM**

## How Many of Each Kind???

### Linear Programming Constraints and Objective Function



Type of Cookie	Amount of Cookies (dozens)	Unit Amount of Dough (lbs per dozen)	Total Dough (lbs)	Unit Amount Icing (lbs per dozen)	Total Icing (lbs)	Unit Prep Time (hrs per dozen)	Total Prep Time (hrs)	Unit Profit (\$ per dozen)	Total Profit (\$)
Plain	x	1	x	0	0	0.1	0.1x	1.50 (\$6.00-4.50)	1.5x
Iced	y	0.7	0.7y	0.4	0.4y	0.15	0.15y	2.00 (\$7.00-5.00)	2y
Limits/ Objectives	$\leq 140$	-----	$\leq 110$	-----	$\leq 32$	-----	$\leq 15$	-----	Maximize!

**Constraints:** Amount of Cookies:  $x + y \leq 140$   
(oven capacity)

Total Dough:  $x + 0.7y \leq 110$

Total Icing:  $0.4y \leq 32$

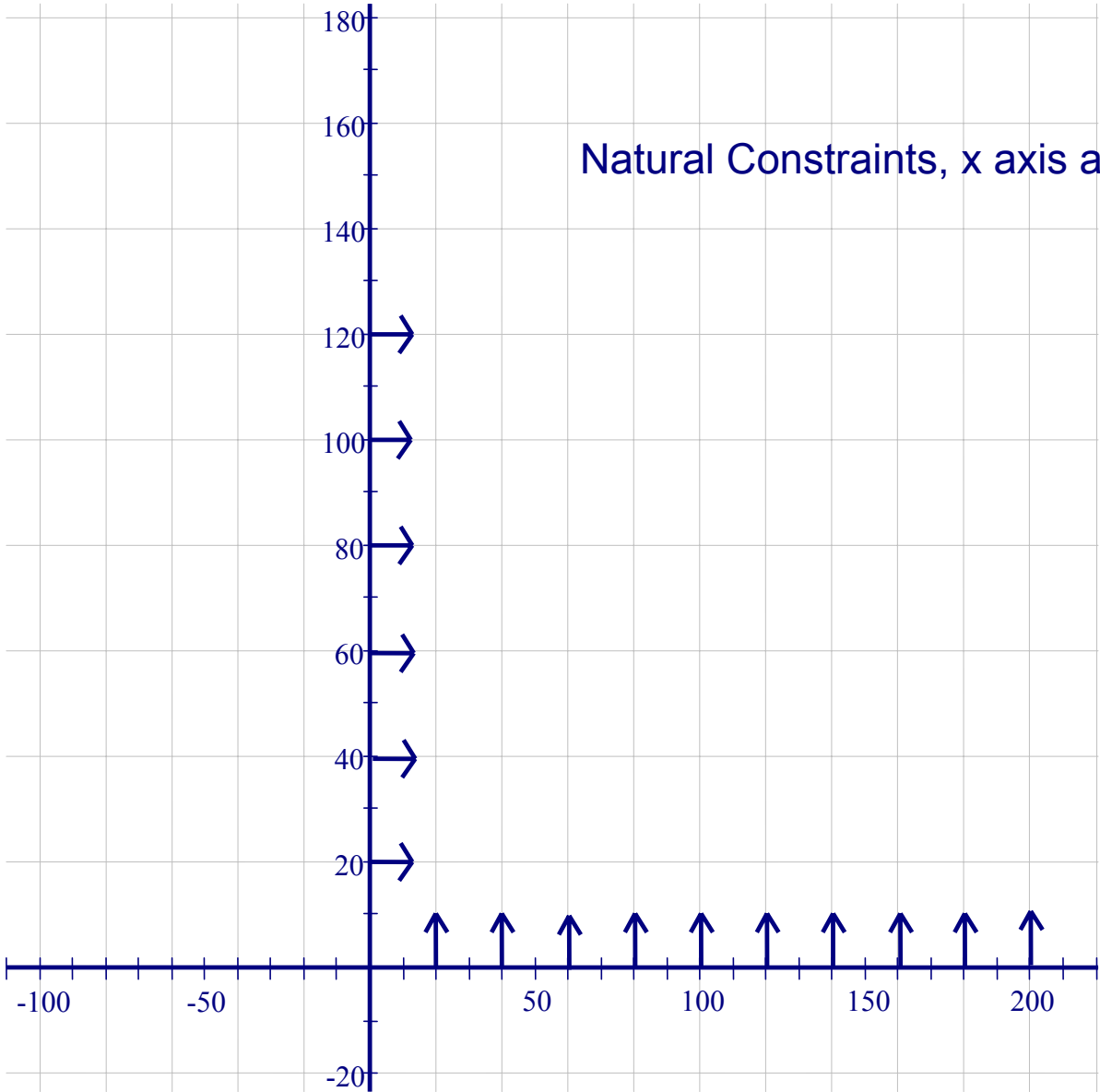
Total Prep Time:  $0.1x + 0.15y \leq 15$

**Objective Function:**

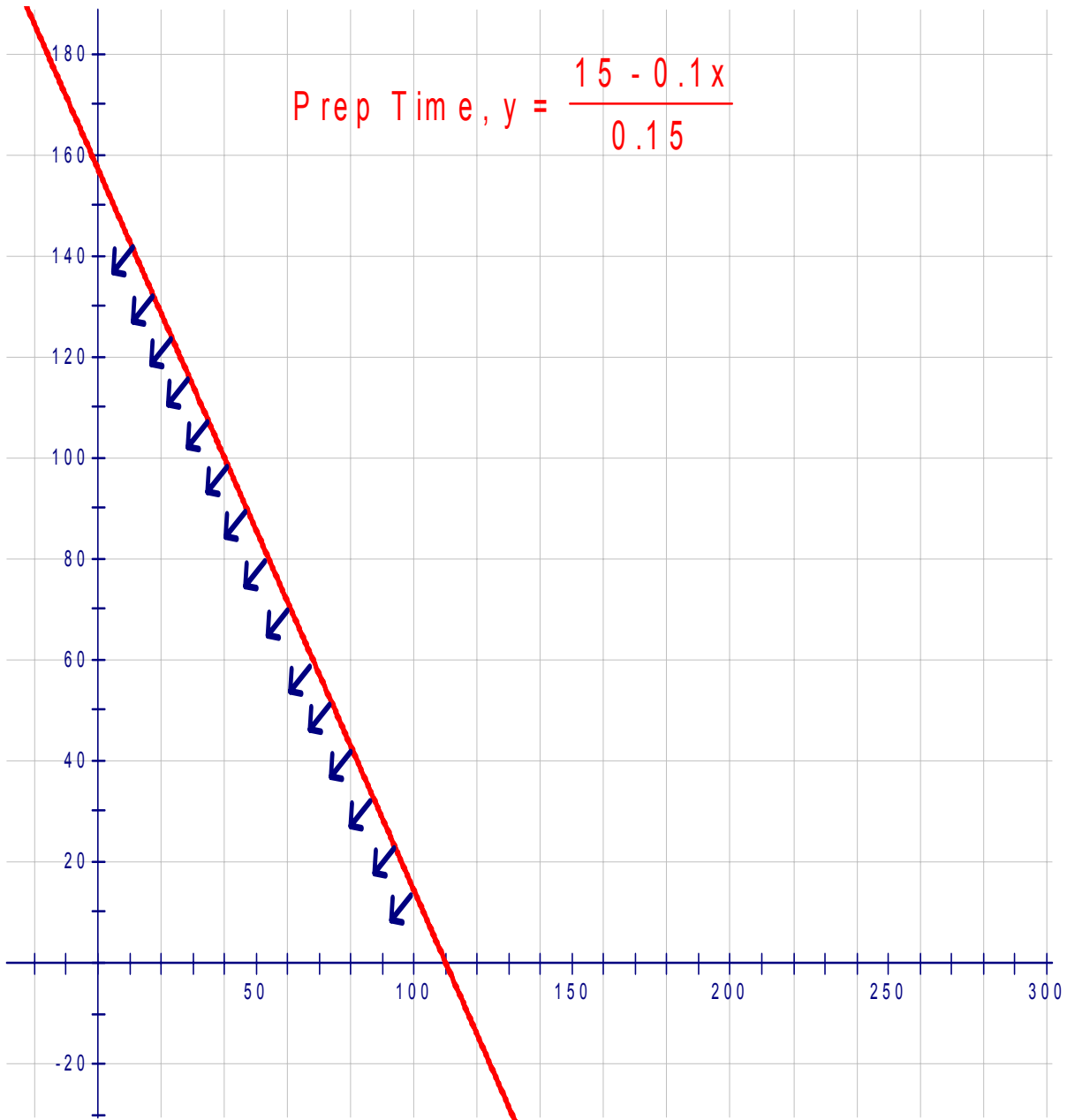
Profit(x, y) =  $1.5x + 2y$   
Maximize!

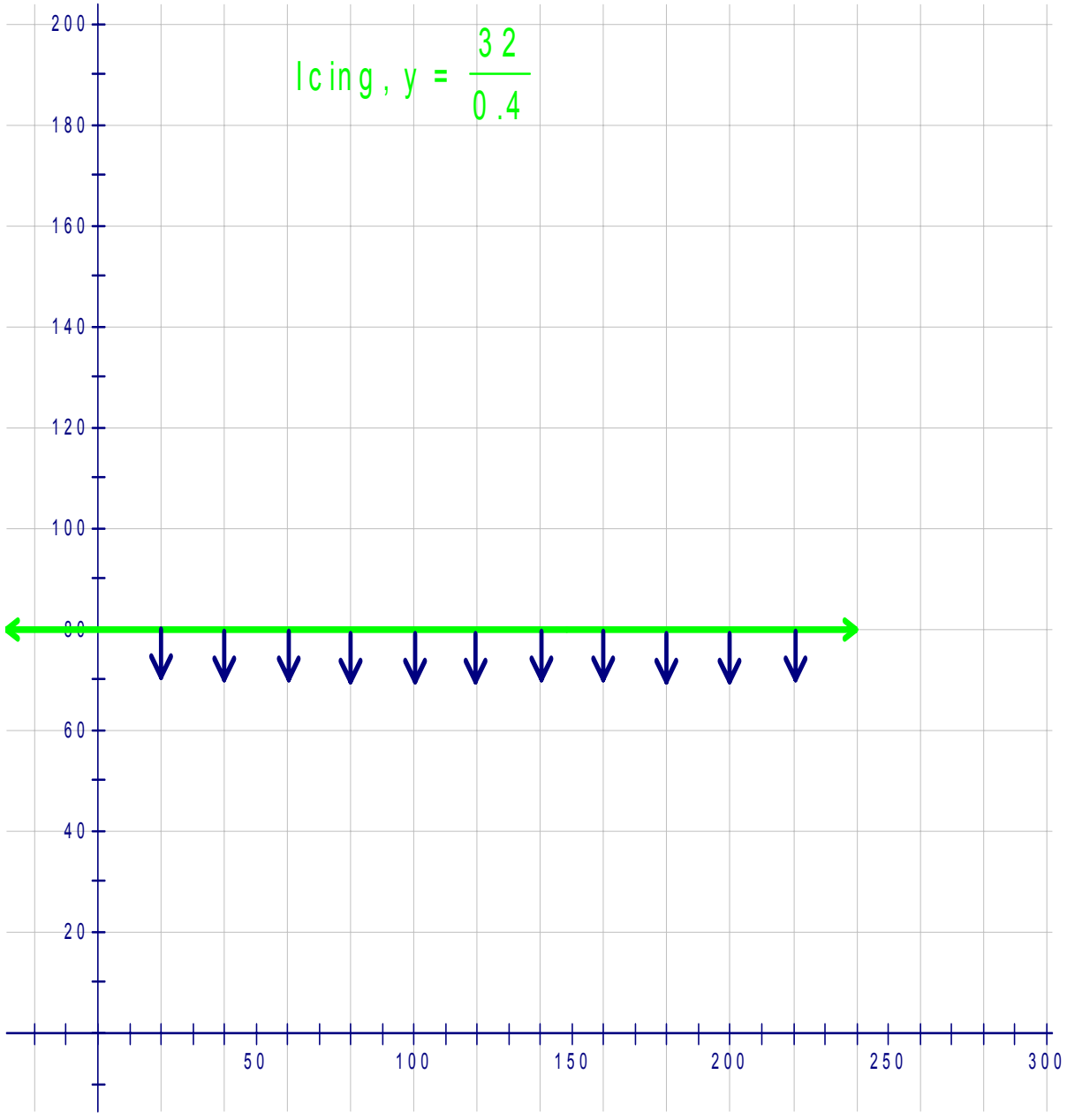
**Natural Constraints:**  $x \geq 0; y \geq 0$  (cannot produce negative amount of cookies)

Natural Constraints, x axis and y axis

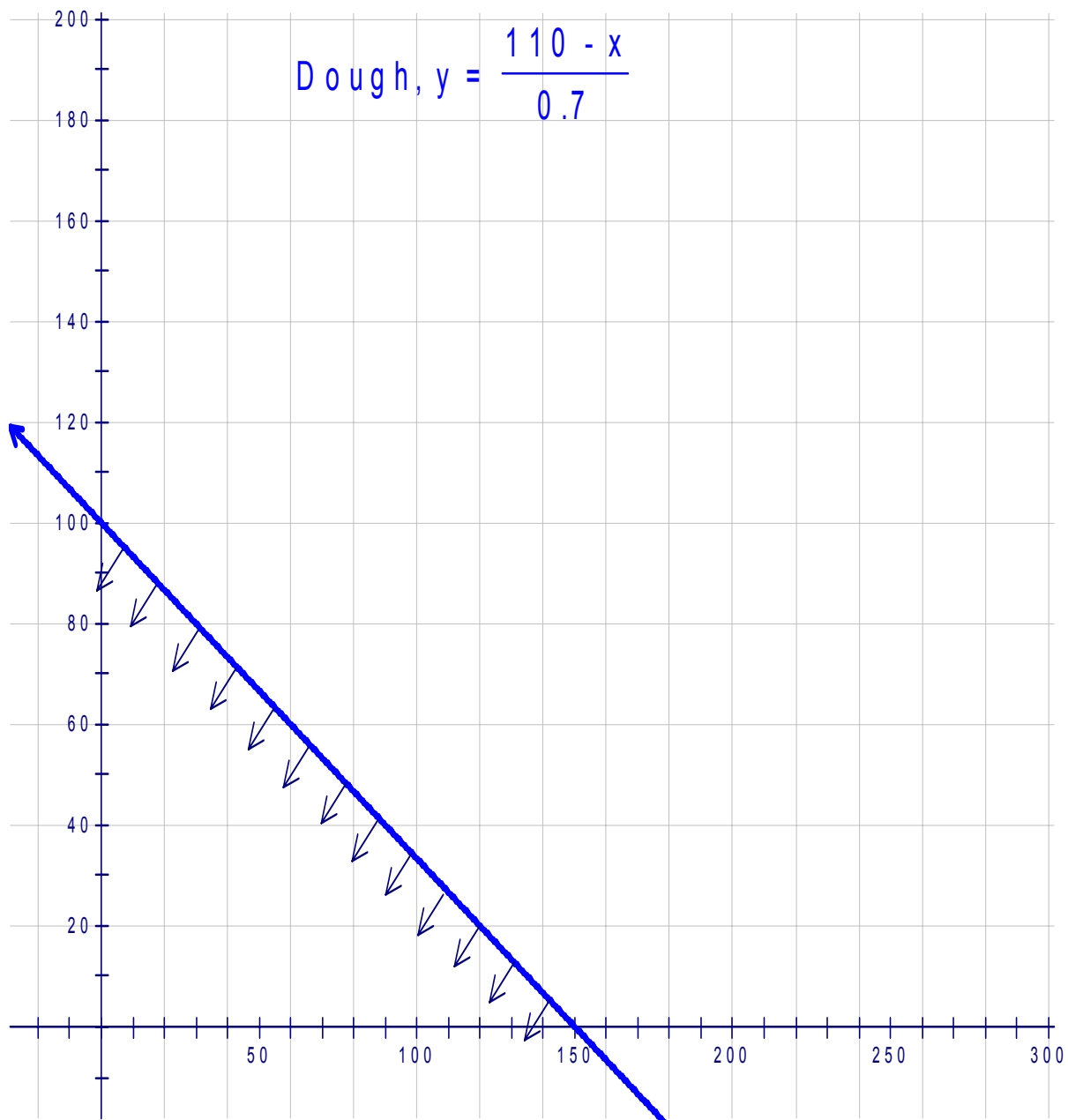


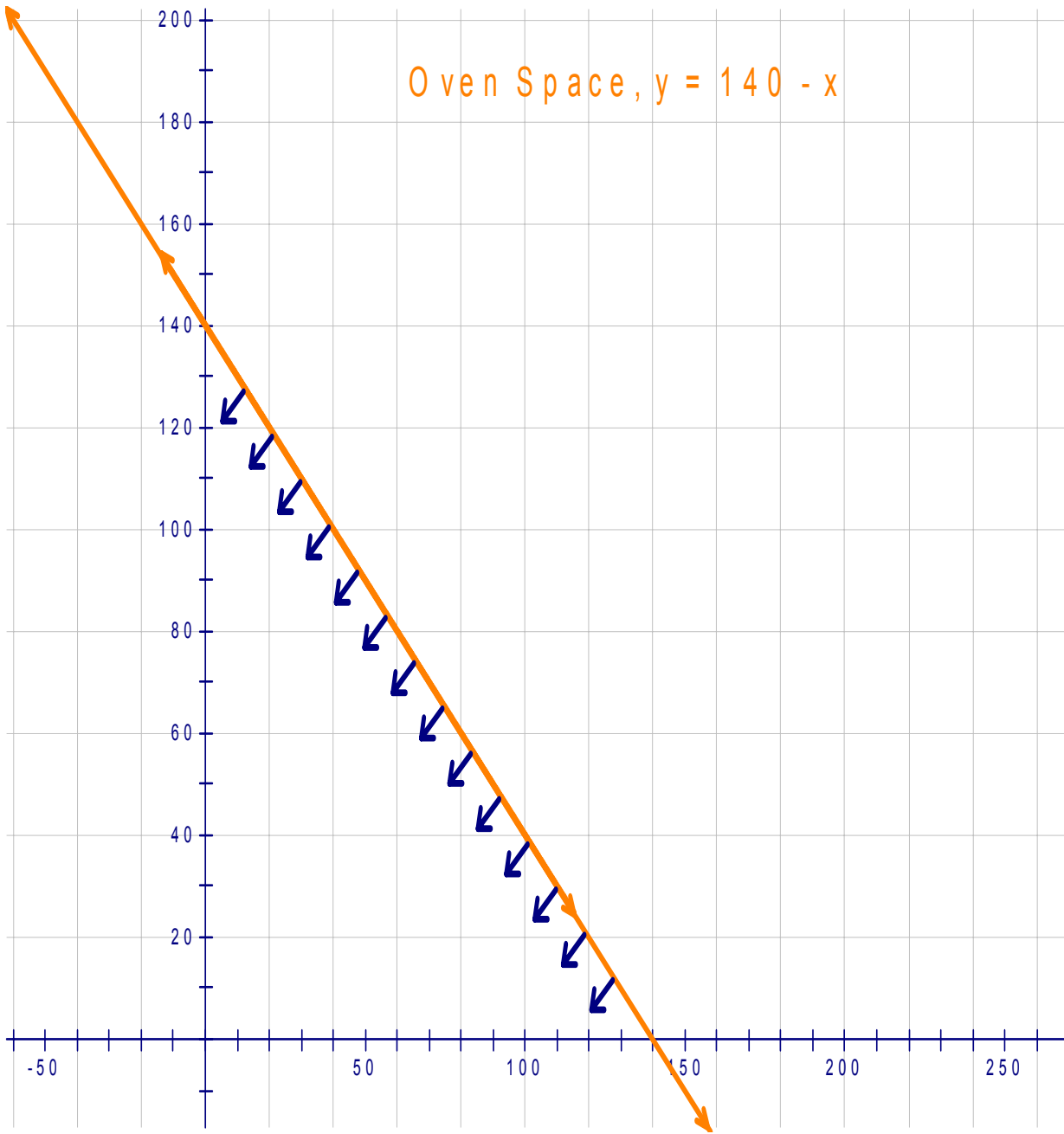
$$\text{Prep Time, } y = \frac{15 - 0.1x}{0.15}$$

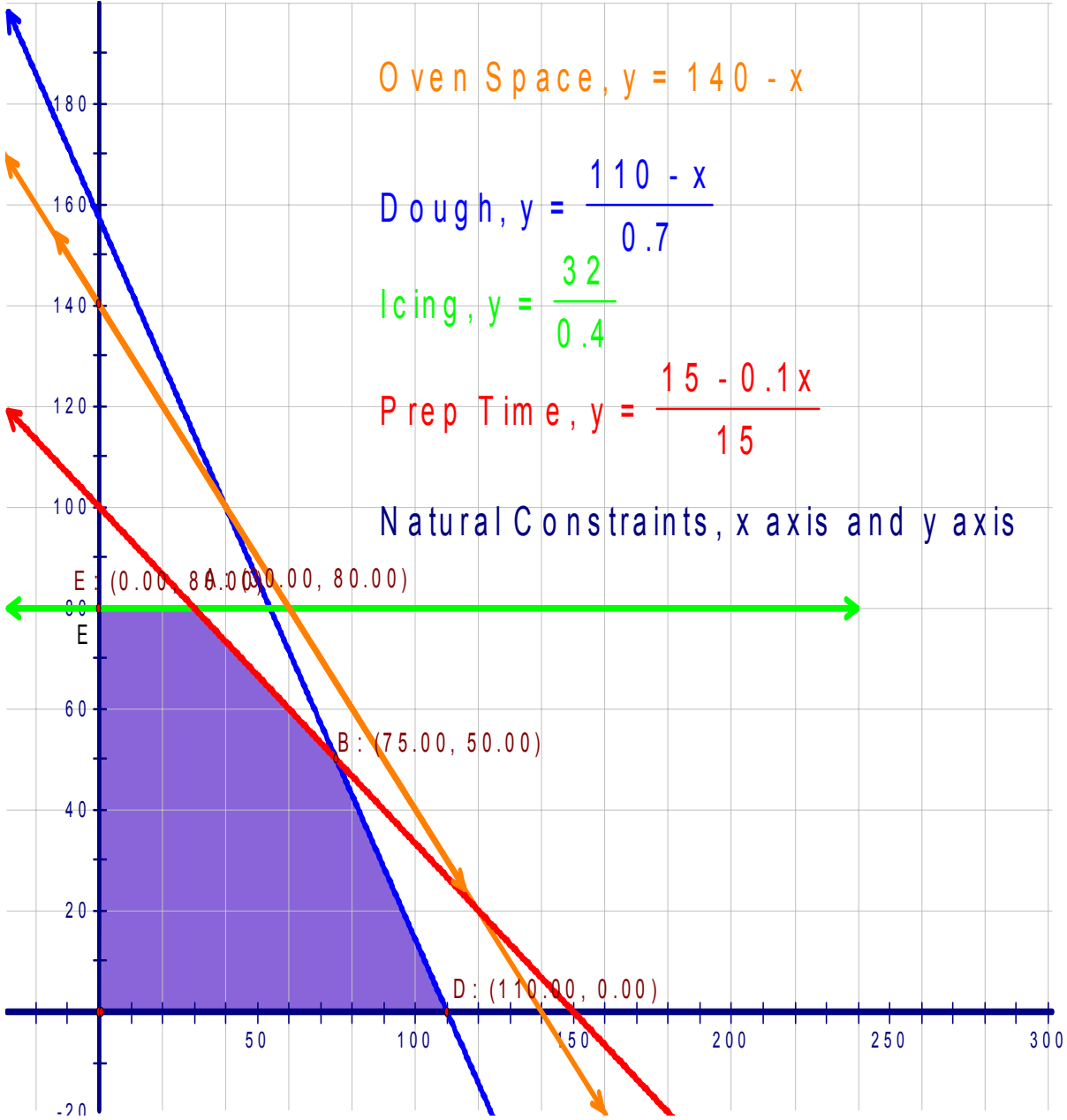




$$\text{Dough, } y = \frac{110 - x}{0.7}$$







Oven Space,  $y = 140 - x$

Dough,  $y = \frac{110 - x}{0.7}$

Icing,  $y = \frac{32}{0.4}$

Prep Time,  $y = \frac{15 - 0.1x}{15}$

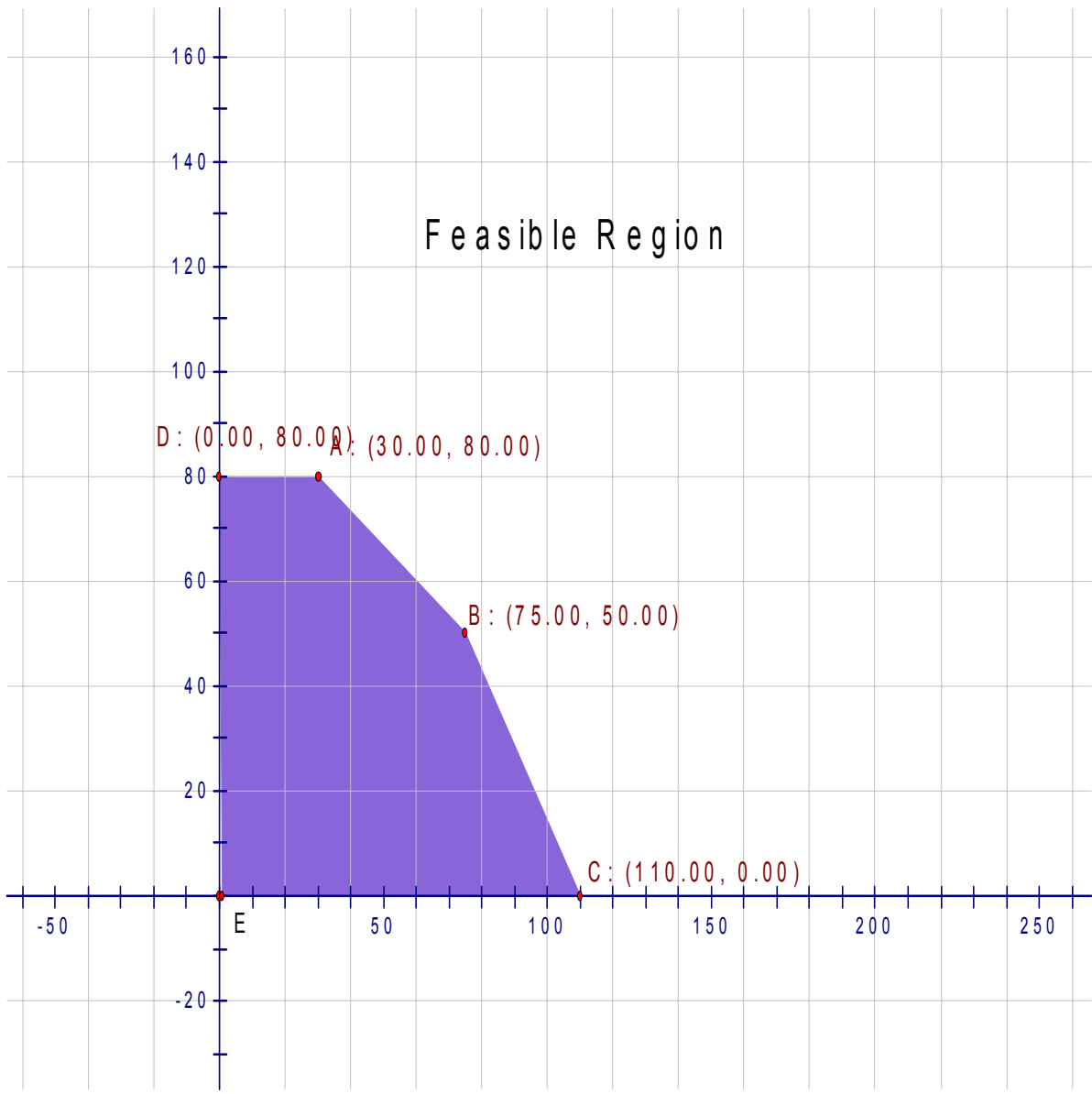
Natural Constraints, x axis and y axis

E: (0.00, 80.00)

A:

B: (75.00, 50.00)

D: (110.00, 0.00)



**ANSWER KEY: ORIGINAL CLASSWORK PROBLEM**

**How Many of Each Kind?**

*Where's the MAXIMUM Profit???*



Corner Point (x, y)	Objective Function $\text{Profit}(x, y) = 1.5x + 2y$	Value of Objective Function at Corner Point
(0,0)	$1.5(0) + 2(0)$	\$0
(0,80)	$1.5(0) + 2(80)$	\$160.00
(30,80)	$1.5(30) + 2(80)$	\$205.00
(75,50)	$1.5(75) + 2(50)$	\$212.50
(110,0)	$1.5(110) + 2(0)$	\$165.00

**Your Conclusions?**

We decided that making 75 dozen plain cookies and 50 dozen iced cookies would be the best combination for the Woo Bakery. This would maximize their profit to \$212.50 per day. They will make 125 dozen altogether (140 was the max oven space), they will use all 110 pounds of dough, 20 pounds out of 32 pounds of icing, and they will use all 15 hours of their prep time.

# How Many of Each Kind?

## Calculator Solution for Original Classwork Problem



The **Y=** screen with corresponding constraints (not including  $x = 0$ ).

```

Plot1 Plot2 Plot3
▼Y1=(110-X)/.7
▼Y2=32/.4
▼Y3=140-X
▼Y4=(15-.1X)/.15

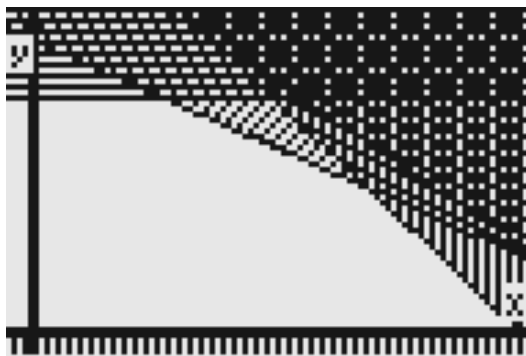
▼Y5=0
▼Y6=
    
```

The **WINDOW** settings screen

```

WINDOW
Xmin=-5
Xmax=110
Xscl=1
Ymin=-5
Ymax=110
Yscl=1
Xres=1
    
```

Graph of constraints (unshaded area is the feasible region)



Substituting one of the corner points into the Objective Equation on the home screen

```

50→I
75→P
2I+1.5P
                    50
                    75
                    212.5
                    212.5
    
```

**Corner Points:** Coordinates found using Intersection option in CALC:

(0,0), (0,80), (30,80), (75,50), (110,0)

**Conclusion:** The point (75, 50) maximizes the Objective Equation. Therefore, 75 dozen plain and 50 dozen iced cookies should be baked in order to

achieve a maximum profit of \$212.50..

**ANSWER KEY: EXTENSION OF CLASSWORK PROBLEM**

## How Many of Each Kind?

*Where's the MAXIMUM Profit???*



Corner Point (x, y)	Objective Function Profit(x, y) = 1.5x + 2y	Value of Objective Function at Corner Point
(0,0)	1.5(0) + 2(0)	\$0
(0,90)	1.5(0) + 2(90)	\$180.00
(25,90)	1.5(0) + 2(90)	\$217.50
<b>(85,50)</b>	<b>1.5(85) + 2(50)</b>	<b>\$227.50</b>
(120,0)	1.5(120) + 2(0)	\$180.00

## Your Conclusions

We decided that making 85 dozen plain cookies and 50 dozen iced cookies would be the best combination for the Woo Bakery. This would maximize their profit to \$227.50 per day. They will make 135 dozen altogether (180 was the max oven space), they will use all 120 pounds of dough, 20 pounds out of 36 pounds of icing, and they will use all 16 hours of their prep time.

**ANSWER KEY: EXTENSION OF CLASSWORK PROBLEM**

## How Many of Each Kind???

### Linear Programming Constraints and Objective Function



Type of Cookie	Amount of Cookies (dozens)	Unit Amount of Dough (lbs per dozen)	Total Dough (lbs)	Unit Amount Icing (lbs per dozen)	Total Icing (lbs)	Unit Prep Time (hrs per dozen)	Total Prep Time (hrs)	Unit Profit (\$ per dozen)	Total Profit (\$)
Plain	x	1	x	0	0	0.1	0.1x	1.50 (\$6.00-4.50)	1.5x
Iced	y	0.7	0.7y	0.4	0.4y	0.15	0.15y	2.00 (\$7.00-5.00)	2y
Limits/ Objectives	$\leq 180$	-----	$\leq 120$	-----	$\leq 36$	-----	$\leq 16$	-----	Maximize!

**Constraints:** Amount of Cookies:  $x + y \leq 180$   
(oven capacity)

Total Dough:  $x + 0.7y \leq 120$

Total Icing:  $0.4y \leq 36$

Total Prep Time:  $0.1x + 0.15y \leq 16$

**Objective Function:**

$\text{Profit}(x, y) = 1.5x + 2y$   
Maximize!

**Natural Constraints:**  $x \geq 0; y \geq 0$  (cannot produce negative amount of cookies.)