Lesson Procedures

For use with Knowledge Rating Chart and Cue Cards

Knowledge Rating Chart ("Vocabulary: Coming to Terms") should be handed out as homework prior to class on vocabulary. Student will rate their working knowledge of the concepts to be covered in the lesson.

Students will be given a set of Cue Cards with each of the terms from Knowledge Rating Chart when entering class on following day upon showing teacher completed Knowledge Rating Chart.

Teacher will review the vocabulary by asking students to give their definitions and uses of each term. Teacher should encourage use of "Cue Cards" if student is not able to define and use term during discussion.

For use with Activities 1 & 2

The following text is taken directly from http://illuminations.nctm.org/lessonplans/9-12/lineardata/index.html :

Exploring Linear Data

Table 4.1 displays data that relate the number or oil changes per year and the cost of engine repairs. Activity Sheet 1, *Oil Changes and Engine Repairs*, uses these data to introduce students to modeling with a linear function. To predict the cost of repairs from the number of oil changes, use the number of oil changes as the x variable and engine-repair cost as the *y* variable. The axes should be labeled, including the units (oil changes/year and dollars), and marked so that all the ordered pairs in the table can be plotted.

Table 4.1

Oil Changes Per Year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost Of Repairs	$300	300	500	400	700	400	100	250	450	650	600	0	150

Figure 4.1 displays the data from table 4.1 graphically. The students are asked to visualize a straight line as a representation of the data. Each student should draw a line that seems to "fit" the plotted points. A line that "fits" the points should have the same characteristics as the set of points; it should actually summarize the data. A line drawn this way is called an eyeball-fit line.

Other fitted, such as the median-fit line (Landwehr and Watkins 1986; Shulte and Swift 1986) and the least squares regression line (North Carolina School of Mathematics and Science 1988), can be constructed depending on the level of the students. Students can compare their lines, which should differ only slightly. All the lines should share one characteristic: they slope downward. This activity, so far, is appropriate for a middle grades class as well as a high school class. The concept of slope can be introduced early in a student's school career if it is approached through an activity based on data.

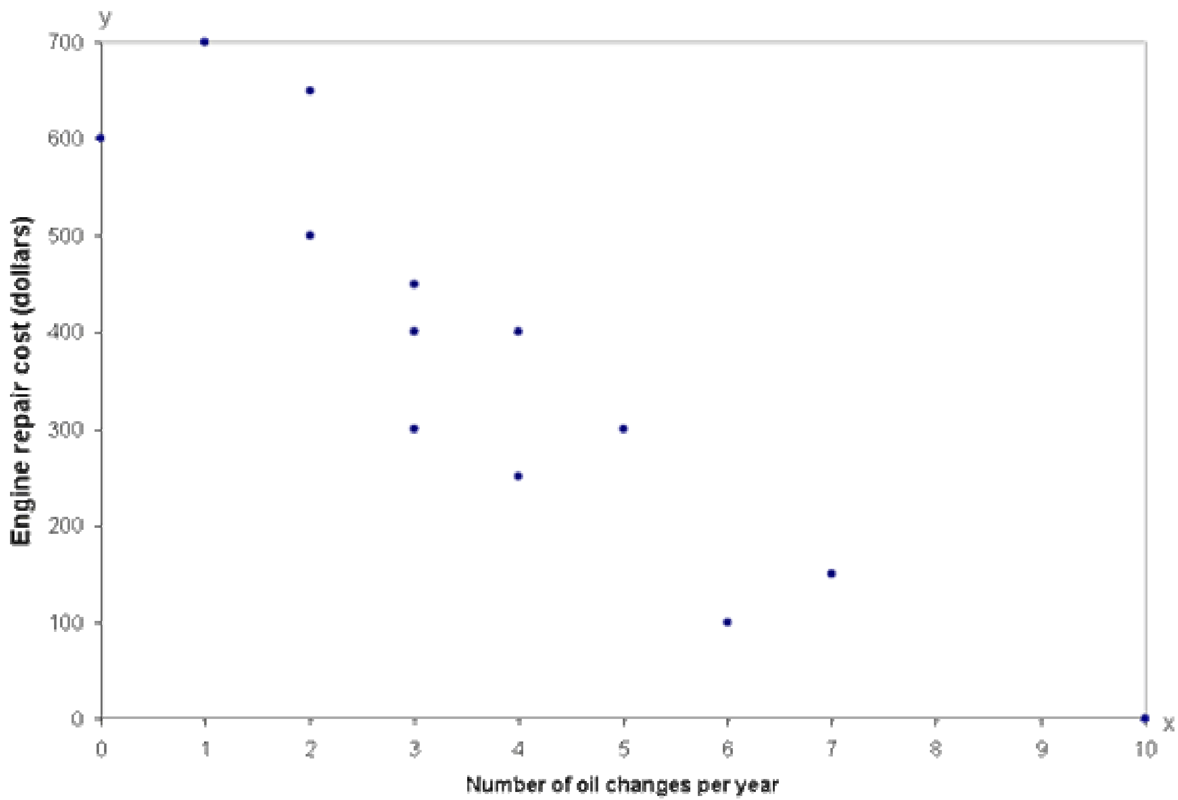


Fig. 4.1

What is the significance of the line's downward (or negative) slope? Students will say it means that the more oil changes made per year, the less money is spent on engine repairs. Note that the correct wording should be, There is a *relation* between the number of oil changes and the money spent on repairs. Although the line represents the data, there is no indication that the number of oil changes causes the need for engine repairs. There are many other variables that affect the cost of engine repairs. More oil changes may just mean those cars have more careful drivers.

Students should examine the slope, which can be found by counting units. First they must define what a unit represents for each variable. In figure 4.1, one horizontal unit represents one oil change. On vertical unit represents 100 dollars in engine repairs. Since students have drawn different lines, the slopes will vary. Note that whereas a fraction or ratio is easily understood as the slope or rate of change, a decimal representation is more useful to compare slopes. After listing all the slopes reported by the class (perhaps in a stem-and-leaf plot), the class should determine a "consensus" slope. The number should be simple to use. For example, -100 is much more useful than -98.73. Since utility is more highly valued than precision in this instance, a slope of -100 seems to be a reasonable answer.

Students should be able to interpret slope as rate of change. What is the change in the cost of repairs for each oil change? A rate of change or slope of -100/1 indicates that for each additional oil change per year, the cost of engine repairs will tend to decrease by 100 dollars. Associating measurement units with the slope is more important to give students a concrete basis for understanding. Students should recognize that changing the units on an axis will affect the slope. If the vertical axis were in cents rather than dollars, the slope would be -10,000.

Next examine the intercepts. The y-intercept is about 750. This means that if there are zero oil changes, engine repairs will cost about $750. From the graph the x-intercept is about 7.5, which means that a car owner would expect to spend nothing on engine repairs if she changed the oil 7.5 times a year. Is this a sensible number of oil changes per year?

The slope and the y-intercept can be used to write the equation of the line:

Writing $y = mx + b$ for $m = -100$ and $b = 750$, we get $y = -100x + 750$.

If the y-intercept is not accessible because of the scale, the equation of the line can be found by using any two points (not necessarily data points) on the line. Students can use the equation to predict the cost of engine repairs expected for a specific number of oil changes. For example, if you change your oil four times a year, how much can you expect to pay in engine repairs? Let $x = 4$, then $y = -100(4) + 750$, or $y = \$350$.

You can expect to pay about $350 in engine repairs.

What is the difference between data points above the line and those below the line? As mentioned in chapter 1, the concept developed here is very helpful for graphing inequalities. Points above the line would indicate that the actual engine repairs exceeded the amount predicted by the number of oil changes. Points below the line represent situations where the engine repairs cost less than predicted. Because the line is only a summary of the relation, just as the mean or median is a summary for a single set of data, there is a degree of variation in using the line to predict. Additional discussion could explore possible reasons, in addition to the natural variation, for this deviation from the line. Excessive engine repairs could be due to bad driving habits; lower than expected repair costs could be due to a special type of oil.

Students can obtain data for further exercises from the newspaper or reference books or use data actually collected from other students. They need to remember that not all data are linear. Before a fitted line is drawn, the most important question to ask is, "Could you imagine a line that represents these points?" Using real data to teach linear equations makes the material relevant and concrete. The last four activities at the end of this chapter provide additional rich contexts for relating linear equations and data.

Notes:

Teaching Matters: It is important that students come to understand the following:

- For some data sets, the assignment of a variable to an axis is purely arbitrary;
- The spread of data about a line can be measured by the correlation coefficient, and the smaller the value of the correlation coefficient, the more important it is to reverse the regression to make an accurate prediction.

Teaching Matters: Initially students can make an eyeball line; later they can find a median-fit line or determine a regression line by using graphing calculator, spreadsheet, or data analysis software.

Teaching Matters: Limitations of using the "best fit" line to predict future data should be discussed. For example, a best-fit line for the winning Olympic times in the 100-meter dash would predict a negative winning time within 20, but of course this is impossible. Nonlinear data are discussed in the next chapter.

Teaching Matters: Emphasize that the intercepts of a fitted line do not make sense for all problems. For example, in Activity Sheet 2, *Bike Weights and Jump Heights*, when the height is 0, the weight cannot be a positive number! Be sure students consider the domain and range implied by the problem context.

Try This: Have students measure their height and the height of their waist from the floor. Plot the data and analyze the linear relation.

Internet Extensions

K - 12 Statistics

Despite the name of this resource, it primarily focuses on grades 9-12, providing an excellent collection of links to lesson plans and other resources, organized using the standard on Statistics for 9-12 from the 1989 NCTM Standards. However, most of these resources are still applicable to the 2000 Standards, and while a few of the links do not work, many excellent resources can be found. Look at "Guessing Correlations" and "Linear Regression".
http://illuminations.nctm.org/swr/review.asp?SWR=348

Rice Virtual Lab in Statistics

The site includes an area that contains all kinds of simulations and demonstrations that pertain to statistics; most are Java applets so you need at least Java 1.1 in order to view the demonstrations. Look for "Regression by Eye."
http://illuminations.nctm.org/swr/review.asp?SWR=340

For use with Activity 3

"How much will they pay?"

For the third activity, students will plot data from a class survey and analyze the relationships among variables. The survey asks what price each student would be willing to pay to watch their favorite music band live in concert. This activity is a variation of the first two activities in that the resulting graph may not be linear. Students will expand their knowledge of graphing beyond linear relationships, but will be able to see relationships in the data despite this change. The teacher must be flexible with this activity as student responses to the survey might be completely different than what is expected.

Activity 3 – Instructional Procedures

(This activity could be used as guided practice or as an independent assessment.)

1. The teacher should survey each student by asking the following question:

"How much would you be willing to pay to see your favorite band live in concert?"

- The students should write their answers in confidence, not knowing what other students write for their answers. This is to encourage an honest, uninfluenced response.
- If class numbers are low, students may be assigned to ask a parent or relative what his or her response would be to that question and bring the answer to school tomorrow.
- The optimum number of responses would be 30-50.

2. Whether during the first or second class period for this activity, the teacher should collect and sort the responses on the chalkboard to present to the class.

- Hand out "Activity 3 Worksheet" and instruct students to complete the first chart using the survey information.
- Students may require assistance understanding the chart and how to input the information.

3. Students complete Activity 3 as shown on the worksheet.

Activity Sheet Masters

Vocabulary: Coming to Terms

Directions: Rate the following terms using the rating scale as shown:

Rating Scale

1. I've never heard the word before
2. I've heard the word, but I don't know how it applies to mathematics
3. I understand the meaning of this word and can apply it to a math problem

Horizontal _____	Plotting Points ___	Equation _________
Vertical _____	Coordinate ________	Regression Line____
X-Axis _______	Minimum _________	Y-intercept ______
Y-Axis _______	Maximum __________	Slope __________

Cue Cards

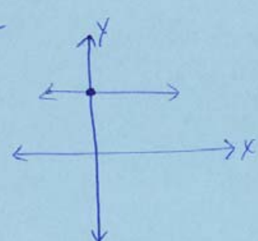
Graphing Calculator

A calculator that will store and draw the graphs of several functions at once.

Horizontal

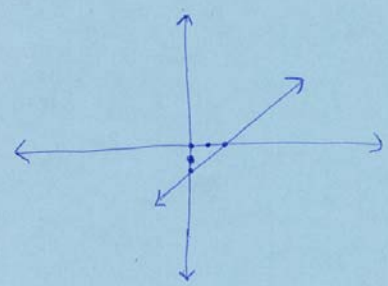
Running from side to side, left to right, right to left...

Y-intercept



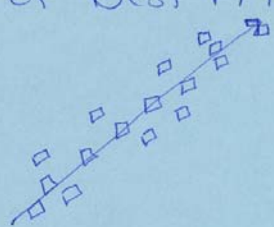
The y-coordinate of the point at which the graph of an equation crosses the y-axis.

Slope



The constant "m" in the linear function equation; rise/run.

Regression Line
or
Line of Best Fit



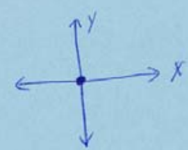
A line that represents the least deviation from the points in a scatterplot of data.

Linear Function

$y = -4x + 1$

A function whose general equation is $y = mx + b$, where m and b stand for constants and $m \neq 0$.

Origin

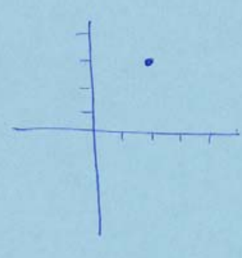


The point at which the two axes intersect.

Ordered Pair
(2, -7)

(x, y) coordinates

Plotting Points
(2,3)



Locating points by means of the coordinates (where the x and y axis intersect.

Equation

$y = 3x - 2$

A statement of equality between two mathematical expressions.

Minimum

0

-14 2 -60

6

The least number in a set of data.

Maximum

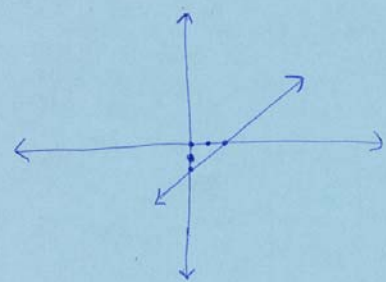
64

47 43

78 16

The greatest number in a set of data.

Slope



The constant "m" in the linear function equation; rise/run.

Regression Line
or
Line of Best Fit

A line that represents the least deviation from the points in a scatterplot of data.

Linear Function
$y = -4x + 1$

A function whose general equation is $y = mx + b$, where m and b stand for constants and $m \neq 0$.

Activity 1
Oil Changes and Engine Repair

1. The table gives data relating the number of oil changes per year to the cost of car repairs. Plot the data on the grid provided, with the number of oil changes on the horizontal axis.

Oil Changes Per Year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of Repairs	300	300	500	400	700	400	100	250	450	650	600	0	150

2. Are the data linear? If so, draw a best-fit line.

3. Find the slope of the line. Describe in words what the slope represents.

4. Find the x- and y-intercepts. Explain in terms of oil changes and engine repairs what each represents.

5. Write the equation of the line.

6. Use the equation to predict the cost of engine repairs if the car had four oil changes. How accurate do you think your prediction is? Explain your answer.

Activity 2
Bike Weights and Jump Heights

1. In BMX dirt-bike racing, jumping high or "getting air" depends on many factors: the rider's skill, the angle of the jump, and the weight of the bike. Here are data about the maximum height for various bike weights. Plot (weight, height). If the data are linear, draw a trend or best-fit line.

Weight (pounds)	Height (inches)
19.0	10.35
19.5	10.30
20.0	10.25
20.5	10.20
21.0	10.10
22.0	9.85
22.5	9.80
23.0	9.79
23.5	9.70
24.0	9.60

2. Is there positive, negative, or no association between bike weight and jump height? Explain your answer.

3. As the weight increases, the height __________________.

4. Find the slope or rate of change. What does this mean in words?

5. Predict the maximum height for a bike that weighs 21.5 pounds if all other factors are held constant.

Activity 3
Attendance at a Concert

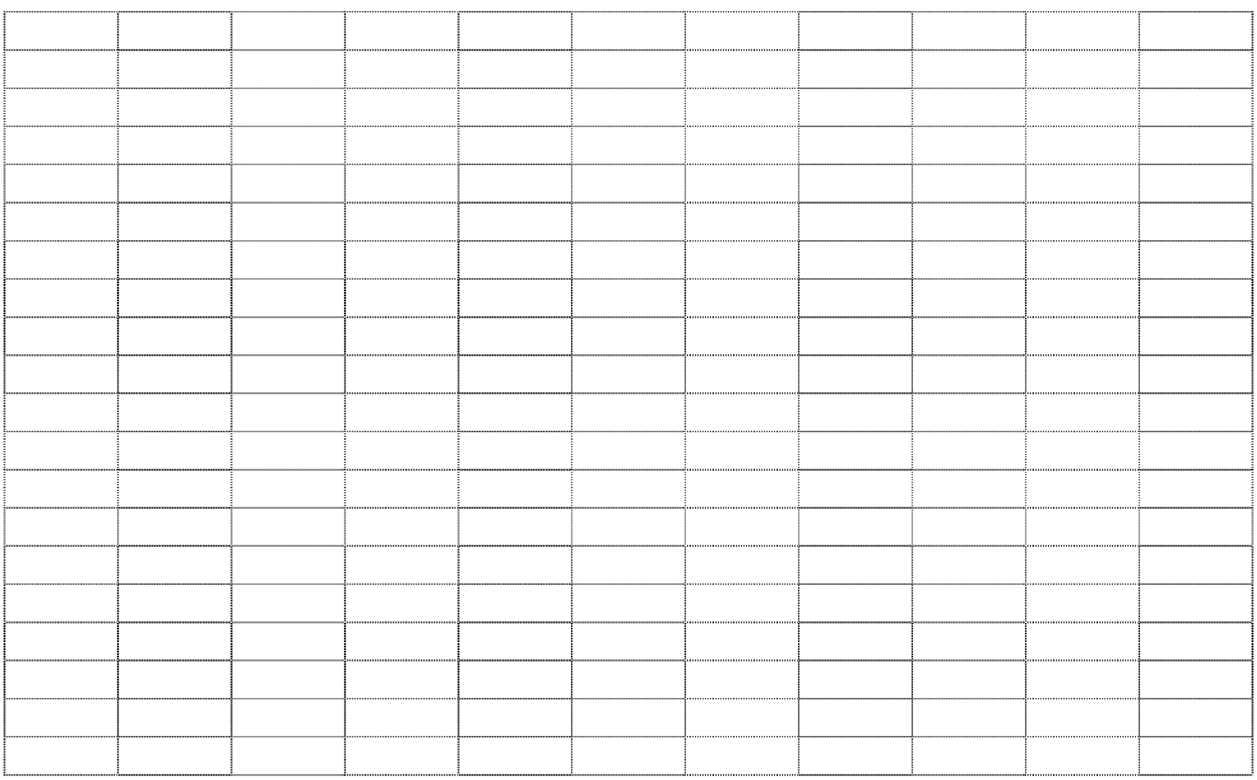
How much would you be willing to pay to see your favorite band give a concert? In this activity, you will discover the relationship between the ticket price for a concert and how many people would be willing to pay a certain price to see it.

Directions:

- Copy the survey results from class into the chart below by writing how many of your friends said they would be willing to pay each price. (Hint: If someone says they would pay $30, assume that they would also pay $0, $10, or $20.)

Price of Ticket ($)	$10	$20	$30	$40	$50	$60	$70	$80	$90	$100
Number of People Who Would Pay										

- Plot these pairs (price, number of people) on the plane below. Put ticket price on the x-axis and number of attendees on the y-axis.
- Answer the questions on page #2 about the data you have collected.



Questions:

1. As the price per ticket increases, the attendance ___________________.

2. What is the relationship between the price of tickets and the number of people who would attend? (circle your answer)

   A. positive relationship

   B. negative relationship

   C. no association

3. Explain your answer to #2.

4. Do the points of this graph appear to describe a linear relationship? ________

5. Is the slope (rate of change) generally constant among these points? ________

6. At what price would you expect the attendance to be zero? (Write answer to the nearest ten dollars.)

Extensions:

1. If you were hosting this concert, what price should you charge per ticket in order to make the highest profit? (Use your graph and consider dollar amounts in only multiples of ten.)

2. What would your maximum profit be?

Activity Sheet Answer Keys

Activity 1 Answer Key
Oil Changes and Engine Repair

2. The table gives data relating the number of oil changes per year to the cost of car repairs. Plot the data on the grid provided, with the number of oil changes on the horizontal axis.

Oil Changes Per Year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of Repairs	300	300	500	400	700	400	100	250	450	650	600	0	150

2. Are the data linear? If so, draw a best-fit line.
They appear to be linear.

3. Find the slope of the line. Describe in words what the slope represents.
The slope is approximately -100.

4. Find the x- and y-intercepts. Explain in terms of oil changes and engine repairs what each represents.
The x-intercept is about 7.5 and the y-intercept is about $750.

5. Write the equation of the line.
y = -100x + 750

6. Use the equation to predict the cost of engine repairs if the car had four oil changes. How accurate do you think your prediction is? Explain your answer.

You would expect to pay about $350.

Activity 2 Answer Key
Bike Weights and Jump Heights

1. In BMX dirt-bike racing, jumping high or "getting air" depends on many factors: the rider's skill, the angle of the jump, and the weight of the bike. Here are data about the maximum height for various bike weights. Plot (weight, height). If the data are linear, draw a trend or best-fit line.

Weight (pounds)	Height (inches)
19.0	10.35
19.5	10.30
20.0	10.25
20.5	10.20
21.0	10.10
22.0	9.85
22.5	9.80
23.0	9.79
23.5	9.70
24.0	9.60

2. Is there positive, negative, or no association between bike weight and jump height? Explain your answer. *Negative*

3. As the weight increases, the height *decreases.*

4. Find the slope or rate of change. What does this mean in words? *-.15; for every 1-pound increase in weight, the height decreases slightly less than 2/10 of an inch.*

5. Predict the maximum height for a bike that weighs 21.5 pounds if all other factors are held constant. *A 21.5-lb bike would be able to jump about 10 inches.*

Activity 3 – (Example Answer Key)
Attendance at a Concert

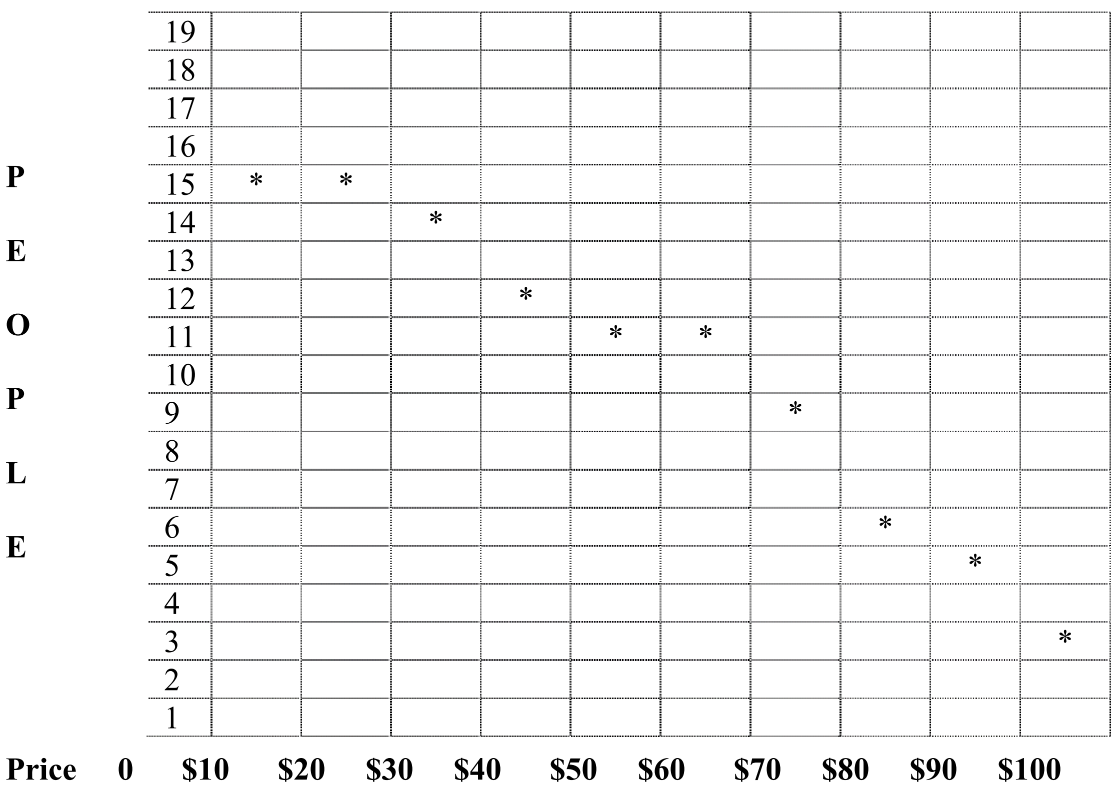
How much would you be willing to pay to see your favorite band give a concert? In this activity, you will discover the relationship between the ticket price for a concert and how many people would be willing to pay a certain price to see it.

Directions:

- Copy the survey results from class into the chart below by writing how many of your friends said they would be willing to pay each price. (Hint: If someone says they would pay $30, assume that they would also pay $0, $10, or $20.)

Price of Ticket ($)	$10	$20	$30	$40	$50	$60	$70	$80	$90	$100
Number of People Who Would Pay	***15***	***15***	***14***	***12***	***11***	***11***	***9***	***6***	***5***	***3***

- Plot these pairs (price, number of people) on the plane below. Put ticket price on the x-axis and number of attendees on the y-axis.
- Answer the questions on page #2 about the data you have collected.



Questions:

1. As the price per ticket increases, the attendance ***decreases*.**

2. What is the relationship between the price of tickets and the number of people who would attend? (circle your answer)

   A. positive relationship

   **** B. negative relationship***

   C. no association

3. Explain your answer to #2.

   The graph gradually goes down as the price goes up.

4. Do the points of this graph appear to describe a linear relationship? ***No.***

   (Students may say "yes" depending on the data.)

5. Is the slope (rate of change) generally constant among these points? ***No.***

   (Answer would be "yes" if relationship appears linear.)

6. At what price would you expect the attendance to be zero? (Write answer to the nearest ten dollars.)

   $110-$130 (This is an estimate in this example.)

Extensions:

1. If you were hosting this concert, what price should you charge per ticket in order to make the highest profit? (Use your graph and consider dollar amounts in only multiples of ten.)

   Profit = Price * Attendance

   **** P($60) = $60 * 11 = $660 ($60 is the best price in this case.)***

   P($70) = $70 * 9 = $630

   P($80) = $80 * 6 = $480

   P($90) = $90 * 5 = $450

2. What would your maximum profit be? ***<u>$660</u>***

Example Survey

Student Responses ("The most I would pay is…")

$20, $25, $30, $40, $60, $60, $70, $70, $75, $80, $90, $90, $100, $100, $100