Mathematics Governor's Institute 2003
Tessellations Problem-in-a-Bag

Title of Project: **"Tantalizing Tessellations"**

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Grade Level and/or Course: Middle School Geometry

Concept(s) used:
Tessellations, transformation, coordinate graphs, measuring, problem solving, and communication

PA Standard(s) Addressed:

2.5.8 Mathematical Problem Solving and Communication

A. Invent, select, use, and justify the appropriate methods, materials, and strategies to solve problems.
C. Justify strategies and defend approaches used and conclusions reached.
D. Determine pertinent information in problem situations and whether any further information is needed for solution.

2.8.5 Algebra and Functions

H. Locate and identify points on a coordinate graph system.

2.8.8 Algebra and Functions

G. Represent relationships with tables or graphs in the coordinate plane and verbal or symbolic rules.
J. Show that an equality relationship between two quantities remains the same as long as the same change is made to both quantities.

2.9.5 Geometry

G. Create an original tessellation
K. Analyze simple transformations of geometric figures and rotations of line segments.

2.9.8 Geometry

C. Classify familiar polygons as regular or irregular up to a decagon.
I. Generate transformations using computer software.
J. Analyze geometric patterns (e.g. tessellations, sequences of shapes) and develop descriptions of the patterns.
K. Analyze objects to determine whether they illustrate tessellations, symmetry, congruence, similarity, and scale.

NCTM Standard(s) Addressed:

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and other contexts.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate mathematical thinking coherently to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.

Introduction / Applications:

Students will be competing in a school wide competition to develop the background design for the school website. Designs should be a tessellation developed in accordance with specific guidelines. The winning design will receive the highest score based on the rubrics below. In the event of a tie score, tie-breaking procedure will be determined at the discretion of the teacher.

Question:

Given a set of guidelines, how will you create your own Escher like tessellation and graph it on coordinate plane?

Model:

The student will create a tessellation that rotates or translates and will explain components of their work. Students will also make generalizations about changing coordinates due to transformations of the fundamental region.

Resources and Materials:

Cardstock
Scissors
Scotch Tape
Crayons/Colored Pencils/Markers
Graph paper
Ruler
Protractor
Construction paper in assorted colors

It is expected that these resources are readily available as part of standard classroom materials.

Procedures & Activities:

1. As an introduction, review transformation vocabulary and coordinate graphing by way of class discussion and development of concept web.

2. Teacher will model creation of a simple tessellation while students follow along at their seats. Teacher will also demonstrate placement of fundamental region on graph paper.

3. Distribute directions, materials, and student activity pages to begin independent work at their seats.

4. Following completion of the tessellations, allow students to share their work with classmates and discuss their strategies.

Answers / Rubric:

This problem has multiple accurate answers. The score for this problem is divided into three categories: Tessellation, Graphing, and Process Writing. Each category is worth five points, for a total of fifteen points for the assignment.

Tessellation

5 points	The student creates a design that tessellates by translation and/or rotation. All images in the tessellation are identical and the tessellation fills the specified amount of space. All parallel sides of the original shape are utilized to create a unique fundamental region. The fundamental region is of appropriate size, based on an original shape of not more than 3 inches by 3 inches. The tessellation has a name and is labeled as such. The student demonstrates high quality and neat work that is fully colored and includes exceptional detail.
4 points	The tessellation meets the criteria listed above (5 points) with the exception of *one* of the following: o Original shape is the wrong size, o The tessellation is not fully colored, o The tessellation is missing "exceptional detail," or o The tessellation is missing a name and label.
3 points	The tessellation meets the criteria above (4 points) with the exception of *two* of the following: o Original shape is the wrong size, o The tessellation is not fully colored, o The tessellation is missing "exceptional detail," or o The tessellation is missing a name and a label. **OR** The student uses only one side of parallel sides of the original shape to develop a unique fundamental region.
2 points	The student generates an image for a fundamental region that does not tessellate, or Only one side of parallel sides is used to develop the fundamental region.
1 point	The design does not tessellate or the original shape is not adjusted at all to create a unique fundamental region (For example, the student uses squares as the fundamental region). The design is not colored.
0 points	Project is not submitted, or no visible student effort is made to complete the tessellation component of the project.

Graphing

5 points	At least ten points of each figure are correctly identified with *(x,y)* coordinates. All points on each figure are labeled using accurate notation (For example, A, A', A", or A*). The problem is completed in Quadrant I. All four images are present on the graph. Student work is neat and readable.
4 points	The graph meets the criteria listed above (5 points) with the exception of *one* of the following: o At least eight points per figure are accurately identified, o The problem is not completed in Quadrant I, or o At least ¾ of the points on the graph are labeled using accurate notation.
3 points	At least three accurate figures are present on the graph. Six or more points are accurately identified on each figure. More than ½ of the points on the graph are labeled using accurate notation. Student work is sloppy.
2 points	At least two figures are present on the graph. At least four points on each figure are accurately identified and labeled.
1 point	Less than four points per figure are accurately identified and labeled. One figure appears on the graph. No labels are present.
0 points	Project is not submitted or no visible student effort is made to complete the graphing component of the project.

Process Writing

5 points	The student accurately identifies and explains the transformation used in the tessellation, using appropriate math vocabulary. The student accurately identifies and explains whether or not the fundamental region used in the tessellation is a polygon. The student identifies meaningful general pattern based on observation of movement of images and corresponding changes to the *(x,y)* coordinates. (For example, the student is able to identify that an increase of 4 in the y coordinate causes the figure to move up 4 units.) The student lists at least 5 real world examples of tessellations. Student writing is neat and free of spelling or grammatical errors.
4 points	The student project meets the criteria listed above (5 points) with the exception of *one* of the following items o The student provides a thorough explanation of questions related to transformations, polygons, and patterns but does not use appropriate math vocabulary, OR o The student lists four real world examples of tessellations, OR o Student writing includes some spelling and/or grammatical errors that do not interfere with the answer's readability.
3 points	The student project meets the criteria listed above (4 points) with the exception of *one* of the following items: o The student is missing or provides an inaccurate response to a question related to transformations, polygons, or patterns, OR o The student lists 3 real world examples of tessellations.
2 points	The student project meets the criteria listed above (3 points) with the exception of *one* of the following items: o The student provides an incomplete or inaccurate response to two questions related to transformations, polygons, or patterns, OR o The student lists 2 real world examples of tessellations, OR o The students' sloppiness and/or spelling and/or grammatical errors interfere with the answer's readability.
1 point	The student attempted to answer the questions but did not provide any accurate answers or conclusions or the student attempted to answer half of the questions assigned for the paragraph. No real world examples of tessellations are provided.
0 points	Project is not submitted or no visible student effort is made to complete the process-writing component of the project, or student work is illegible.

Accommodations/Adaptations

ESL:

- Review vocabulary and create word webs for coordinate graph(ing) terms (in addition to those related to tessellations).
- Use illustrations to contrast *regular* and *nonregular* polygons.
- Use a mirror to help students understand *reflection*.

Special Ed:

- Instead of asking students to develop their own shapes for fundamental regions, allow students to select from a collection of shapes that will tessellate.
- Allow students to graph just one image of their fundamental region instead of three.
- Use illustrations to contrast *regular* and *nonregular* polygons.
- Use a mirror to help students understand *reflection*.

Enrichment/Extension:

- Direct students to complete their tessellations using a computer program.
- Graph and analyze coordinates of transformations on a computer or graphing calculator.

Have students explore and answer the following questions, orally or in a journal entry or written report.

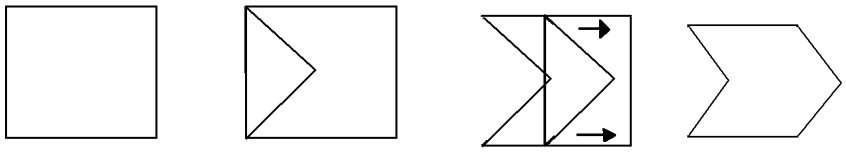
- Using a four quadrant grid, create a reflection of your fundamental region over the x and y axes. What are your new coordinates?
- Research and write a short report about M.C. Escher. Explain why these projects are called "Escher-like tessellations."
- Does your tessellation have any lines of symmetry? Rotation symmetry? How do you know?
- Which of the first twelve regular polygons (triangle, quadrilateral, pentagon…) tessellate? What determines whether or not they tessellate? Describe your conclusions in a general pattern.
- Explore tessellations in the world around you. Take pictures of tessellations that you see in architecture and nature and create a photo scrapbook of your findings.

Directions to Create Your Own Tessellation!

I. Color one side of a shape that will tessellate.

II. Make a fundamental region using one of the following methods:

Slide Method
Cut a portion of a fundamental region on one side, slide it to the opposite side, and tape it. (This is usually easier if you start at a vertex!) Repeat: You can do the same thing with any other pair of parallel sides.



Rotation Method
Cut a portion of your fundamental region on one side and rotate it around one of its vertices so that the straight sides line up, then tape. Repeat this process with any other pair of parallel sides.

III. Use your fundamental region to make a tessellation on the paper provided. Do this by tracing the fundamental region, moving it, tracing it in a new location and repeating this process until your page is filled.

IV. Add detail using colored markers, crayons, etc.

V. Cut around the outer edge of your finished paper and mount it on large construction paper.

VI. Name and label your tessellation.

VII. Write your name on the bottom right hand corner.

VIII. Staple your fundamental region to your tessellation on the bottom left corner.

Name:_____

Graphing Tessellations
Student Activity Page

1. On the first quadrant of a coordinate graph, make a tessellation by tracing your fundamental region four times.
2. Label your fundamental region and its images as "Fundamental Region" and "Image I," "Image II," and "Image III."
3. Choose at least ten points that would make a rough outline of your fundamental region and label them alphabetically. Label the corresponding points on Images I, II, and III as A', A'', and A*, respectively.
4. Complete the chart below by finding the coordinates of each of the points on your figures. You will use this table to look for patterns and make generalizations later in the project.

List the coordinates of your images below:

Fundamental Region			Image I			Image II			Image III		
	X	**Y**		**X**	**Y**		**X**	**Y**		**X**	**Y**
A			**A´**			**A″**			**A***		
B			**B´**			**B″**			**B***		
C			**C´**			**C″**			**C***		
D			**D´**			**D″**			**D***		
E			**E´**			**E″**			**E***		
F			**F´**			**F″**			**F***		
G			**G´**			**G″**			**G***		
H			**H´**			**H″**			**H***		
I			**I´**			**I″**			**I***		

Name: _____

Process Writing—Final Assignment

Write a paragraph (or more) in the space below that addresses the following issues:

- Is the fundamental region of your tessellation a polygon? Why or why not? Explain how you know.
- Explain all transformations used in your tessellation. How do you know that they are transformations?
- Study your list of coordinates (from the tessellation on graph paper). Generalize any patterns that you see in the changing coordinates and the movement of your images.
- Outside of math class, where might you see tessellations in your world? Please list at least five examples.

Transformation Concept Web