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Grade Level: 9-12

Content Area: Statistics and Data Analysis

PA Standard(s) addressed:

2.6.11.A …Describe the data as an example of a distribution using statistical measures of center…

NCTM Standard(s) addressed:

Select and use appropriate statistical methods to analyze data.

Problem Name: The Wage War

Problem:

A local union's contract is just about up and negotiations have begun. The most important issue for each side is wages. The company's representative as well as the union's representative has each submitted an average wage to justify their side of the argument on this issue. The funny thing is that the numbers do not match. Here are the wages for the fourteen union workers: $5.00, $8.00, $11.00, $15.00, $18.00, $18.00, $18.00, $19.00, $21.00, $22.00, $22.00, $23.00, $29.00, $31.00.

Directions:

Solve the following questions showing all work involved. Explain each step and justify your answers. Remember you must show all the steps you used to solve the problem even if you use a calculator. To receive the highest score, all calculations and steps must be shown and explained in writing. Numeric answers must be labeled.

a. Which average wage (mean, median or mode) did the company report? What was the number they used? Why did they pick that average?
b. Which average wage (mean, median or mode) did the union report? What was the number they used? Why did they pick that average?
c. If two additional workers were hired during negotiations, what wages would the workers have to be paid to end the wage war (Hint: equalize the mean, median and mode)?

Problem Solution(s):

To solve the problem you must find the mean, median and mode of the wages. To do this you have to sort the data in ascending order.

$5, $8, $11, $15, $18, $18, $18, $19, $21, $22, $22, $23, $29, $31

The median and mode can now be found by looking at the set. The mode is $18 because it occurs most frequently. The median is the middle number. Since there is an even

number of data points, the median is found by averaging the two middle numbers \$18 and \$19. The median is \$18.50. $\left(\frac{18+19}{2}=18.5\right)$

The mean is found by adding all the data points together and dividing by however many data points there are. The mean is approximately \$18.57.

$$\left(\frac{5+8+11+15+18+18+18+19+21+22+22+23+29+31}{14}\approx 18.57\right)$$

Therefore, mean = \$18.57, median = \$18.50 and mode = \$18.00.

The answer to question a. could be several responses. The student can choose any of the three averages, but he/she must back up their response with a correct justification. Example: I picked the mean because it is the highest average. The mean is \$18.57. This would help the company to show that the union workers make more money and deserve a smaller raise on the next contract.

The answer to question b. could also be several responses. The student can choose any of the three averages, but he/she must back up their response with a correct justification. Example: I picked the mode because it is the lowest average. The mode is \$18.00. This would help the union to show that the union workers make less money and deserve a larger raise on the next contract.

The answer to question c. could also be several responses. To solve part c. the student has to consider how to end the wage difference. That is simple; make your possible choices all the same so there are no discrepancies. That is, make the mean, median and mode the same. There is one more thing to consider, there are 16 workers now. That means there are two wages that we need to make up. Consider the wages:

\$5, \$8, \$11, \$15, \$18, \$18, \$18, \$19, \$21, \$22, \$22, \$23, \$29, \$31

The mode is \$18 and unchangeable since there are only two of any other number. Adding another number to the one double (\$22) would make two different modes. Therefore the number we want the mean, median and mode to be is \$18. Since the mean has to be \$18, the two missing wages would have to total \$28. This is true because the sum of the sixteen numbers must total 16 times \$18 which is \$288. Since the sum of the given fourteen wages is 260, the remaining two wages must add up to 28.

$$\text{mean} = \frac{\text{sum of the wages}}{\text{total number of wages}}$$

Therefore:

$$18 = \frac{5+8+11+15+18+18+18+19+21+22+22+23+29+31+\_\_+\_\_}{16}$$

$$288 = 5+8+11+15+18+18+18+19+21+22+22+23+29+31+\_\_+\_\_$$

$$28 = \_\_+\_\_$$

What are the two wages? Does it matter? It does. If you look at the list of wages given, the two new wages can be placed within the set in three different ways. Both wages can be placed to the right of the median, one on each side of the median or both on the left of the median.

5, 8, 11, 15, 18, 18, 18, 19, 21, 22, 22, 23, 29, 31

↑

median

Case 1: Both are to the right of the median.

If both wages are placed on the right of the median, the wage must be at least $19. The sum of the smallest two wages, $19 and $19, would equal $38, which is larger than the necessary sum of $28. Therefore both numbers cannot both be on the right of the median.

Case 2: One on each side of the median.

If one wage is placed on each side of the median, the median will remain $18.50. Therefore this will not work because we need it to be $18.

Case 3: Both are to the left of the median.

If both wages are placed on the left of the median, any two wages whose sum is $28 would work. The possible solutions would fall into the following set: {(18,10), (17.99,10.01), (17.98, 10.02), …, (14.02,13.98), (14.01,13.99), (14,14)}.

Specific Rubric:

5. Advanced Understanding:

- Correct answers to all three parts with correct procedures/correct calculations shown and described and a written explanation that describes the work. Correct understanding and use of mean, median, and mode should be demonstrated.

4. Satisfactory Understanding:

- Correct answers to all three parts with correct procedures/correct calculations shown and described and a written explanation that describes the work. Correct understanding and use of mean, median, and mode should be demonstrated.
- May have a minor omission in calculation or explanation.

3. Almost Satisfactory Understanding:

- Correct answers to all three parts with most correct procedures/correct calculations shown and described and no written explanation that describes the work. Some steps are missing, but work can be followed.
- Correct answers to all three parts with few correct procedures/correct calculations shown and described and some written explanation that describes the work.
- Incorrect answer with correct procedures/correct calculations shown and described and some written explanation, but with one error carried through.

2. Partial Understanding:

- Correct answers to all three parts with few correct procedures/correct calculations shown and described and some written explanation that describes the work.
- Incorrect answer with most correct procedures/correct calculations shown and described and little written explanation.
- Incorrect answer with correct procedures and no explanation. May have no more than two calculation/copying errors

1. Minimal Understanding:

- Correct answers to all three parts with calculations, procedures or explanation that are not understandable or missing.
- No answer or an incorrect answer, but the student has provided some information relevant to calculating the solution.

0. No Understanding:

- Blank or off task work.