

Names: Teri Edwards, Marilyn Goldfarb, Ron Ploucha, Robert VanHorn

Grade Level: 9-12 **Content Area:** Geometry/Trigonometry

PA Standard(s) addressed: 2.9.11. I

NCTM Standard(s) addressed: Use visualization, spatial reasoning, geometric modeling to solve problems.

Problem Name: “*Shapes R Us*”

Problem: A government contract has just been awarded to Shapes R Us, an aluminum fabricating company. The contract calls for the production of circles with a diameter of 5 inches. Shapes R Us was successful in obtaining this contract because of the company’s commitment to recycling. They have created a special machine that not only cuts the circles but also automatically recycles the waste from each sheet of aluminum into a new sheet of aluminum from which more circles can be cut. The production foreman has been instructed to determine the total number of circles possible and the minimum number of recycling operations necessary to completely process one sheet of aluminum measuring 2 feet by 24 feet. His calculations will be based on the following conditions.

1. No aluminum is lost in the recycling process.
2. The machine will ONLY accept sheets of aluminum that measure exactly 2 feet in width.
3. All circles must be complete circles.
4. During the calculation process, all decimals will be rounded to hundredths.

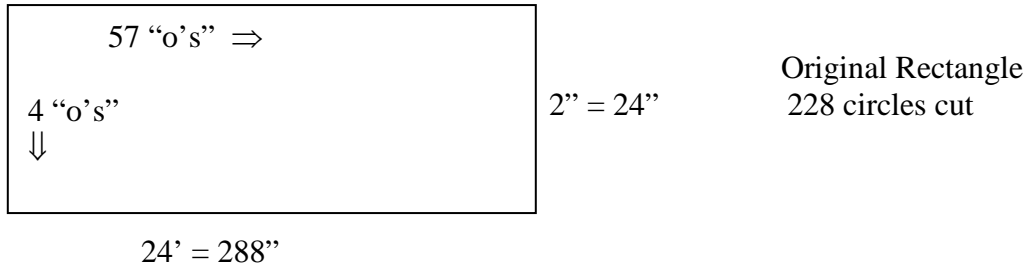
Assuming that the production foreman follows correct procedures and adheres to the conditions set forth, **HOW MANY CIRCLES ARE POSSIBLE AND HOW MANY RECYCLING PROCEDURES ARE REQUIRED** to process the original sheet of aluminum until the length of the recycled sheet will no longer accommodate full circles?

Directions: For full credit, you **must** do the following:

1. Show or describe each step of your work, including formulas and diagrams, even if you did calculations in your head (“mental math”) or used a calculator.
AND
2. Write an explanation stating the mathematical reason(s) why you chose each of your steps.

Problem Solution:

To solve the problem, I first drew the original piece of metal and converted feet into inches to match the diameter of the circle. I then found how many 5” full circles would fit across the length by dividing 288” by 5” which is 57. Similarly, 4 full 5” circles will fit in the width. Therefore, 57x4 or 228 circles can be cut from the original piece of metal.



To find the amount of metal wasted, I found the area of the rectangle and then subtracted the area of the 228 circles.

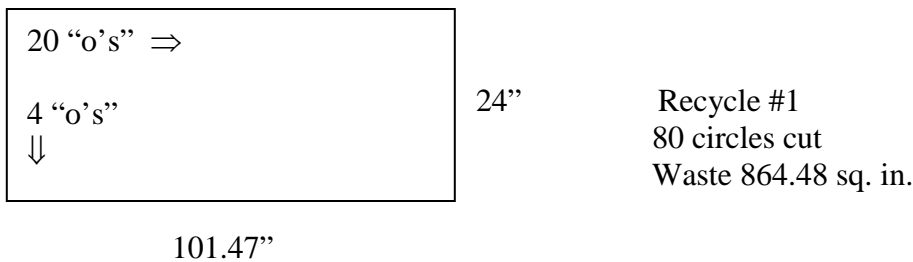
$$\begin{aligned}
 A(\text{Rect}) &= L \times W \\
 &= (288)(24) \\
 &= 6,912 \text{ sq. in.}
 \end{aligned}$$

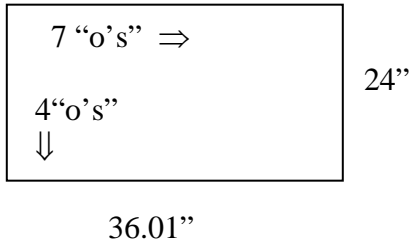
$$\begin{aligned}
 A(\text{Circle}) &= \pi r^2 \\
 &= \pi (2.5)^2 \\
 &= 19.63 \text{ sq. in}
 \end{aligned}$$

$$A(\text{Circle}) \times 228 \text{ circles} = 4,476.77 \text{ sq. in.}$$

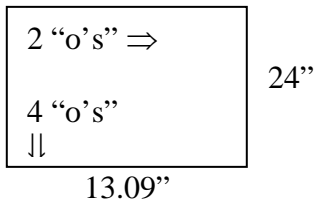
$$A(\text{Rect}) - A(228 \text{ circles}) = 2,435.23 \text{ sq. in. of waste}$$

The next figure represents the total area of the next rectangle (Recycle #1). Since the width of 24” remains constant, to find the length of the new rectangle, I divided 2435.23 sq. in. by 24” to get 101.47”. The exact same process is repeated to find 80 circles and waste of 864.48 sq. in.

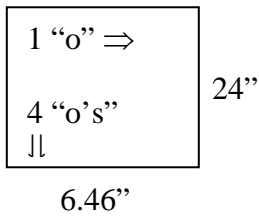




Recycle #2
28 circles cut
Waste 314.21 sq. in.



Recycle #3
8 circles cut
Waste 154.92 sq. in.



Recycle #4
4 circles cut
Waste 76.5 sq. in.

This will be the last recycle because when 76.5 sq. in. is divided by 24" width, the length of the rectangle will be 3.19". This is too small to cut a full 5" diameter circle.

Therefore, the total number of full 5" diameter circles cut will be:

$$228 + 80 + 28 + 8 + 4 = 348 \text{ circles}$$

It will take 4 recycling procedures to do this!

Special note:

Teachers may want to have their students use graphing calculators to explore the data from this problem in a scatter plot (recycle # vs. amount of waste). This problem can be adapted in the following fashion:

Change the dimensions to 2 feet by 500 feet. Ask students to predict the number of recycles needed to complete the task.

Specific Rubric:

5. Advanced Understanding:

- Correct answer (348 circles, 4 recycles)
- All work shown
- Explanation includes how any why steps were performed

* This problem is iterated several times. A student may explain the process of the first iteration, and then state that he/she repeated the process to arrive at the other stated solutions for full explanation credit.

OR

- Incorrect answer with no more than two minor computational errors.
- All work shown
- Explanation includes how any why steps were performed

4. Satisfactory Understanding:

- Correct answer (348 circles, 4 recycles)
- All work shown
- Some explanation is included.

OR

- Incorrect answer with no more than two minor computational errors.
- All work shown
- Some explanation is included.

3. Almost Satisfactory Understanding:

- Correct answer (348 circles, 4 recycles)
- Some work shown
- Some explanation is included.

OR

- Incorrect answer with no more than two minor computational or procedural errors.
- Some work shown
- Some explanation is included.

OR

- Correct answer for first cycle (228 circles, 1 recycle)
- All work is shown for first cycle.
- All explanation for the first cycle is shown

2. Partial Understanding:

- Correct answer (348 circles, 4 recycles)
- Some work shown
- No explanation is included.

OR

- Incorrect answer with no more than five minor computational or two procedural errors.
- Some work shown
- No explanation is included.

OR

- Correct answer for first cycle (228 circles, 1 recycle)
- All work is shown for first cycle.
- No explanation for the first cycle is shown

1. Minimal Understanding:

- Correct answer (348 circles, 4 recycles)
- No work shown
- No explanation is included.

OR

- Incorrect answer
- No work shown
- Some plausible explanation of an approach is included.

OR

- Correct answer for first cycle (228 circles, 1 recycle)
- All work is shown for first cycle.
- No explanation for the first cycle is shown

0. No understanding

- No response

OR

- Off task response

OR

- Does not meet criteria of a 1