

# **Governor's Institute for Mathematics**

**July 29 - August 2, 2002**

## **Creators:**

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**Content Area: Mathematics**

**Grade Level: 9 - 12**

## **PA Standard(s) addressed:**

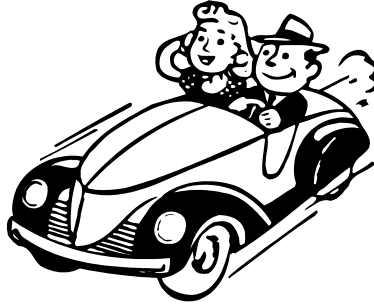
Major Emphasis: 2.2.11.A Develop and use computation concepts, operations, and procedures with real numbers in problem-solving situations.  
2.2.11.C Construct and apply mathematical models, including lines and curves of best fit, to estimate values of related quantities.

Additional Standards: 2.1.11.A, 2.2.11.B, 2.2.11.F, 2.4.11.B, 2.5.11.A, 2.5.11.B, 2.5.11.C, 2.5.11.D, 2.8.11.D, 2.8.11.L, 2.8.11.N, 2.8.11.Q, 2.8.11.R, 2.11.11.C

## **NCTM Standard(s) addressed:**

Number and Operations: Grades 9 – 12

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.



**Problem Name:** Let's Make a Deal

**Problem:**

Ted B. Thrifty is considering selling his 1996 Euclid which is in perfect condition. Its initial value was \$26,000. A local car dealer, Sam Shady tells him that a two year old car is worth \$20,000 and a five year old car is worth \$11,000. With further research, Ted learns from the local car guide that his car depreciates 14% each year and he decides to use this method for his calculations.

This year, Sam Shady has offered Ted \$8000 as a fair deal for the Euclid. Ted declined the offer stating, "Your offer is unfair. After careful calculations, I have decided that you should offer me at least \$10,520."

- A. Why did Ted reject Sam's offer? How did Ted arrive at his counter offer?
- B. At what point in time will Ted's method of calculation always be greater than the dealer's method. Justify your answer.

## Directions:

For full credit, you **must** do the following:

1. Answer the questions and show all the steps you used in solving the problem. If you used a calculator, you must write a complete description of the steps that you followed. If you did some of the work mentally, you must write a complete description of the steps that you followed.

**AND**

2. Write an explanation stating the mathematical reasons **why** you chose each of the steps.



## Problem Solution(s):

### TABLE- METHOD SOLUTION

#### PART A

#### SAM SHADY'S (DEALER) METHOD

Original price of Ted's car = \$26,000

Prices given by Sam Shady

PRICE	\$20,000	\$11,000
YEAR	2	5

Using the prices given by Sam, the decrease per year can be calculated by change in price divided by the change in years.

Sam is decreasing the value of the car by  
 $(\$20,000 - \$11,000) / 3$  years or \$3000 per year.

Using the decrease per year, a table can be constructed showing the value of the car at the end of each year.

YEAR	PREVIOUS VALUE - \$3000	NEW VALUE
1	26,000 – 3,000	23,000
2	23,000 – 3,000	20,000
3	20,000 – 3,000	17,000
4	17,000 – 3,000	14,000
5	14,000 – 3,000	11,000
6	11,000 – 3,000	8,000

### TED'S METHOD

Original price is \$26,000 with a 14% decrease from the previous years value.

YEAR	PRICE
1	\$22,360.00
2	\$19,229.60
3	\$16,537.46
4	\$14,222.21
5	\$12,231.10
6	\$10,518.75

The values in the table above can be calculated using two different methods:

- 1) Multiply previous value of the car by .14 giving you the depreciation amount.  
Subtract the depreciation from the previous year's value.

$$\begin{aligned}\text{Example: } \$26,000 \times .14 &= \$3,640 \\ \$26,000 - \$3,640 &= \$22,360\end{aligned}$$

- 2) Subtract 14% from 100% to calculate the percentage value that the car is currently worth. Convert that percentage to a decimal and then multiply by the previous year's value. Estimate answer to nearest cent.

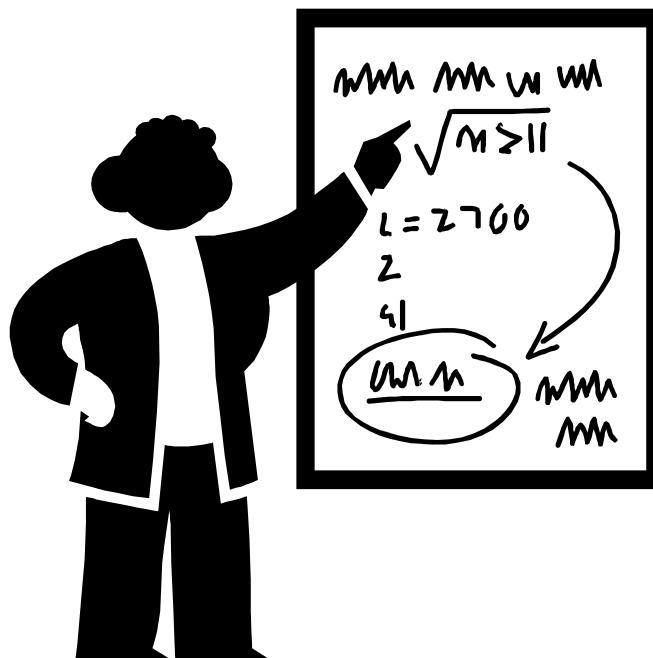
$$\begin{aligned}\text{Example: } 100\% - 14\% &= 86\% \\ 86\% \text{ as a decimal is } &.86 \\ \$26,000 \times .86 &= \$22,360\end{aligned}$$

Since it is 2002, Ted's car is now 6 years old. By looking at the table values for year 6, clearly Sam's offer of \$8,000 is well below the value of \$ 10,518.75 calculated using Ted's depreciation method. Therefore, Ted rejects Sam's offer. He counter-offered by rounding his calculated value to the nearest 10 dollars.

## PART B

By looking at the table and comparing the values at each year, it is noted that Ted's calculated trade in value becomes greater than Sam's at the fourth year.

\*Note: Answer will vary depending on the current date.



# ALGEBRA/GRAPHING METHOD SOLUTION

## PART A

### SAM SHADY'S ( DEALER ) METHOD: LINEAR

- Calculate slope as the change in  $y \div$  change in  $x$ , with the given data (2 yrs. , \$20,000) and (5 yrs. , \$11,000).

Slope =

$$\frac{\$20,000 - 11,000}{2 \text{ years} - 5 \text{ years}} = \frac{\$9,000}{-3 \text{ years}} = \$ - 3,000/\text{yr.}$$

- A slope value of  $-3,000$  means Sam Shady's Method causes the car to lose \$3,000 each year.
- Use the slope and an appropriate formula to calculate an equation for Sam's Method with either  $y = mx + b$  or  $y - y_1 = m(x - x_1)$ .  
Substituting with the point (2 yrs., \$20,000) and  $m = \$ - 3,000/\text{yr.}$

$$20,000 = -3,000(2) + b$$

$$20,000 = -6,000 + b$$

$$26,000 = b$$

Thus, Sam's Method equation is:

$$y = -3,000x + 26,000 \text{ or } y = 26,000 - 3,000x.$$

## OR

- The problem gives the initial value of the car as \$26,000. This can be used as the y-intercept, (0, 26,000) and lead to the same equation as above.

### TED'S METHOD: EXPONENTIAL

- Using the general form for exponential depreciation,  $y = ab^x$ , substitute the values as:

$$Y = \text{Initial Value}(1 - \% \text{depreciation})^x$$

$$Y = 26,000(1 - 0.14)^x$$

Thus, Ted's Method equation is:  $y = 26,000(.86)^x$

- Use a graphing calculator to type Sam's Equation in  $y_1$  and Ted's Equation in  $y_2$  by pressing the Y= key. Use an appropriate window in the first quadrant.

For example,  $X_{\min} = 1$   
 $X_{\max} = 10$   
 $X_{\text{scl}} = 1$   
 $Y_{\min} = 0$   
 $Y_{\max} = 27,000$   
 $Y_{\text{scl}} = 1,000$   
 $X_{\text{res}} = 1$

- To view the graph, push the GRAPH button.
- Ted arrived at his counter offer because it is the year 2002 and his car was a 1996, so he subtracted:  $2002 - 1996 = 6$ , and then substituted  $x = 6$  into his method equation.

$$Y = 26,000(.86)^6$$

$$Y = \$10,518.75$$

Ted estimated this to be \$10,520.00

- We now realize why Ted rejected Sam's offer of \$8,000 and countered with \$10,520.

## PART B

- Find the point of intersection of the two graphs by entering the button sequence, 2<sup>nd</sup>, TRACE, 5. This yields  $x = 3.73$  and  $y = 14,823.04$ .

Therefore, when  $x = 3.73$  years both methods would give \$14,823.14 for the value of the car.

- By viewing the graph in the window, it can be seen that the y-value is greater on the linear graph for x-values at 3.73 and below.
- The graph shows that the y-value is greater on the exponential curve for x-values greater than 3.73.

- Therefore, Ted's method will always be greater after 3.73 years. Estimating this to the nearest whole year, we would get  $x = 4$  years.

\* Note: Answer will vary depending upon the current date.



## Rubric

### Advanced Understanding

- 5 Correct justification for Part A and Correct answer for Part B  
(4 years)  
All work is shown  
Explanation includes how and why the steps were performed

### Satisfactory Understanding

- 4 Correct justification for Part A and Correct answer for Part B  
(4 years)  
All work is shown  
Some explanation is included

**OR**

- Correct justification for Part A and Correct answer for Part B  
(4 years)  
Some work is shown  
Full explanation is included

### Almost Satisfactory Understanding

- 3 Correct justification for Part A and Correct answer for Part B  
(4 years)  
All Work is shown  
No explanation is included

**OR**

- Correct justification for Part A and Correct answer for Part B  
(4 years)  
Some work is shown  
Some explanation is included

**OR**

- Incorrect answer due to calculation error but work process  
leads to correct answer  
Some work is shown  
Some explanation is included

## **Partial Understanding**

**2** Correct justification for Part A and Correct answer for Part B  
(4 years)

Some work is shown

No explanation is included

**OR**

Incorrect answer due to multiple calculation errors

Some work is shown

Some or no explanation is included

**OR**

Incorrect answer due to a procedural error

Some work is shown

Some or no explanation is included

## **Minimal Understanding**

**1** Correct justification to Part A **OR** Correct answer for Part B  
(4 years)

No work is shown

No explanation is included

**OR**

Incorrect justification/answer due to multiple procedural or  
calculation errors

At least 1 procedure that could lead to the solution included

Some or no explanation is included

## **No Understanding**

**0** Blank **OR**

Off task response **OR**

Does not meet requirements for a **1**

We're outta here !!  
See ya !

