Open-Ended Template

Names: Paula Bartkovsky, Susan Morris, Dyvonne Nevil, Johanna Rebarchak

Grade Level: 9-12 **Content Area**: Measurement

PA Standard(s) addressed:
2.3.11 Describe how a change in linear dimension of an object affects its perimeter, area and volume.

NCTM Standard(s) addressed:
Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Name:
The Icky Sticky Candy Company Dilemma

Problem:

The Icky Sticky Candy Company is introducing a new product, Good n' Icky candy. Nine ounces of candy will be packaged in individual boxes with dimensions of 2 cm x 4.5 cm x 6 cm. For shipping ease, a carton is to be designed to hold 6 individual boxes. Considering this, what would be the dimensions of the shipping carton of the least surface area?

Directions:

For full credit, you must do the following:

1. Show all the steps you used to solve the problem. Describe any steps that were completed in your head or with a calculator.

And

2. Write an explanation justifying your answer.

Problem Solution(s):

Solution # 1:

The most suitable packing carton would be the carton with the smallest surface area. All possible dimensions can be determined along with their respective surface areas.

The formula for surface area is Perimeter of the Base times the height + 2 times the Area of the Base.

Dimensions *	**Calculations**	**Surface Area**
2 x 4.5 x 36	(13) (36) + 2 (2) (4.5)	486 cm^2
2 x 27 x 6	(58) (6) + 2 (2) (27)	456 cm^2
12 x 4.5 x 6	(33) (6) + 2 (12) (4.5)	306 cm^2
4 x 13.5 x 6	(35) (6) + 2 (4) (13.5)	318 cm^2
4 x 4.5 x 18	(17) (18) + 2 (4) (4.5)	342 cm^2
2 x 9 x 18	(22) (18) + 2 (2) (9)	432 cm^2
6 x 9 x 6	(30) (6) + 2 (6) (9)	288 cm^2
2 x 13.5 x 12	(31) (12) + 2 (2) (13.5)	426 cm^2

* All dimensions given are in centimeters.

Of the eight possible dimensions (disregarding the order of length, width, and height), the smallest surface area is 288 cm^2, making the 6 cm x 6 cm x 9 cm carton the most suitable.

Solution # 2:

Example #1

4 cm x 13.5 cm x 6 cm
SA = 2 $F_{\#1}$ + 2 $F_{\#2}$ + 2 $F_{\#3}$

FACE # 1 = 13.5 cm x 4 cm = 54 cm^2
FACE #2 = 6 cm x 4 cm = 24 cm^2
FACE #3 = 13.5 cm x 6 cm = 81 cm^2

SA = 2(54 cm^2) + 2(24 cm^2) + 2(81 cm^2)
SA = 318 cm^2

Example # 2

6 cm x 6 cm x 9 cm
SA = 2 $F_{\#1}$ + 2 $F_{\#2}$ + 2 $F_{\#3}$

FACE # 1 = 6 cm x 6 cm = 36 cm^2
FACE #2 = 6 cm x 9 cm = 54 cm^2
FACE #3 = 6 cm x 9 cm = 54 cm^2

SA = 2(36 cm^2) + 2(54 cm^2) + 2(54 cm^2)
SA = 288 cm^2

Example #3

4.5 cm x 6 cm x 12 cm
SA = 2 $F_{\#1}$ + 2 $F_{\#2}$ + 2 $F_{\#3}$

FACE # 1 = 12 cm x 6 cm = 72 cm^2
FACE #2 = 4.5 cm x 6 cm = 27 cm^2
FACE #3 = 4.5 cm x 12 cm = 54 cm^2

SA = 2(72 cm^2) + 2(27 cm^2) + 2(54 cm^2)
SA = 306 cm^2

The dimensions 6 cm x 6 cm x 9 cm produce the smallest surface area of 288 cm^2. Several stacking arrangements are possible including cartons with dimensions 4 cm x 6 cm x 13.5 cm, 4.5 cm x 6 cm x 12 cm, and 6 cm x 6 cm x 9 cm. Next, surface area for all possibilities must be determined. An interesting fact occurred that the closer the carton's dimensions approached those of a cube, the smaller the surface area of the carton. For example, the surface area of the 6 cm x 6 cm x 9 cm was 288 cm^2. The surface area of the 4 cm x 6 cm x 13.5 cm was 318 cm^2. A hypothesis can be made that a carton with dimensions closest to the cube would have the smallest surface area. For example, the surface area of a 4 cm x 4.5 cm x 18 cm carton was 342 cm^2, thereby, supporting the hypothesis. There would be no need for further calculations.

Specific Rubric:

5. Advanced Understanding:

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- All work is shown.
- Explanation includes how and why the steps were performed.

4. Satisfactory Understanding:

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- All work is shown.
- Some explanation is included.

3. Almost Satisfactory Understanding:

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- All work is shown.
- No explanation is included.

OR

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- Some work is shown.
- Some explanation is included.

OR

- Incorrect answer due to a calculation error
- Some work is shown.
- Some explanation is included.

2. Partial Understanding:

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- Some work is shown.
- No explanation is included.

OR

- Incorrect answer due to multiple calculation errors
- Some work is shown.
- Some or no explanation is included.

OR

- Incorrect answer due to procedural error
- Some work is shown.
- Some or no explanation is included.

1. Minimal Understanding:

- Correct answer (Dimensions of the carton: 6 cm x 6 cm x 9 cm)
- No work is shown.
- No explanation is included.
- Incorrect answer due to multiple procedural or calculation errors.
- At least one procedure that could lead to the solution is included.
- Some or no explanation is included.

0. No Understanding:

- Blank

  OR
- Off task response

  OR
- Does not meet requirements for a 1