Fear Factor

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Grade Level: 9 - 12 **Content Area**: Probability and Predictions

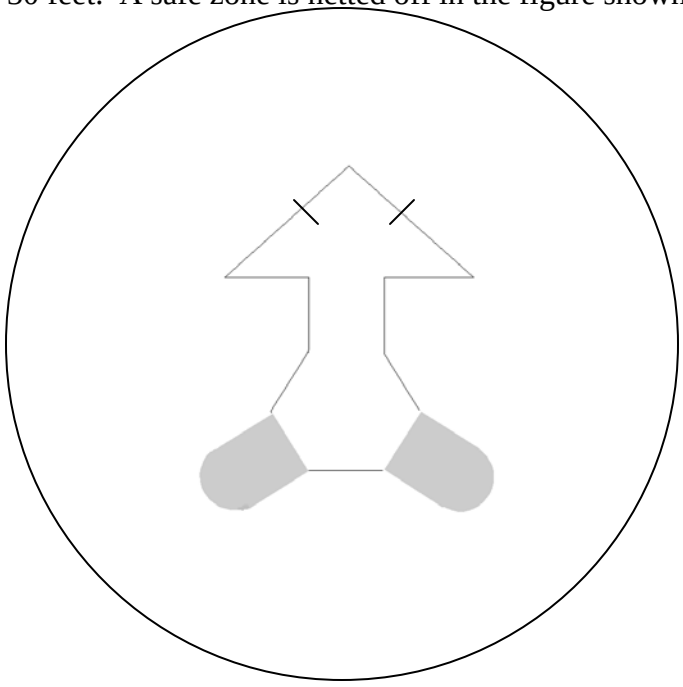
PA Standard(s) addressed:
2.7.11
Use experimental and theoretical probability distributions to make judgments about the likelihood of various outcomes in certain situations.

2.9.11
Use properties of geometric figures to determine areas and calculate geometric probabilities.

NCTM Standard(s) addressed:
Understand meaning of probabilities and how they relate to events and predictions and calculating geometric probabilities.

Problem Name: Fear Factor

Problem: In a Fear Factor challenge, contestants are airdropped into a circular region with a diameter of 30 feet. A safe zone is netted off in the figure shown below.



**Note: All measurements on the diagram are 4 feet except for the legs of the isosceles right triangle at the top of the figure.

A. In Round One, you were one of the 57 contestants dropped. If ten landed in the safe zone, what is the probability that you landed in the safe zone?
B. What is the probability, theoretically, that a given contestant will land in the safe zone?
C. Contestants that land in the shaded region will win an additional $1000 cash bonus. What is the probability of winning the bonus money?
D. Since too many people won the challenge, the network decided to shrink the safety zone so the chances of winning are reduced to being between 7 and 8 percent. Sketch a diagram of a safety zone that meets this requirement and justify your reasons.

Directions:
For full credit, you must do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

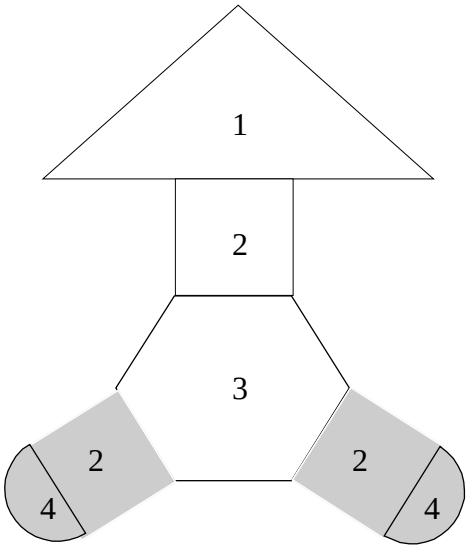
2. Write an explanation stating the mathematical reason(s) **why** you chose each of the steps.

Problem Solution(s):

A. To find the experimental probability of landing in the safe zone, divide the number of successes by the number of attempts.

Probability = $\frac{10}{57}$ = .1754 = 18%

B. To find the theoretical probability of landing in the safe zone, divide the area of the safe zone by the area of the circular region. The area of the safe zone is found by dividing the figure into common shapes. The way a student divides the drawing may vary, however, their explanation of their thinking must include a description of that process. The sum of this area is 138.13 square feet. One possible way of finding that area is as follows:



Area of triangle 1: First find the hypotenuse to be 12 feet. Then use the 45-45-90 rule to determine the legs of the triangle to be $6\sqrt{2}$. Since the legs are the base and the height then,

$A = \frac{1}{2}bh = \frac{1}{2}(6\sqrt{2})(6\sqrt{2}) = 36$ square feet.

Area of squares 2: Since each side is 4 feet and the area of a square is the length of the side squared, then $A=4^2 = 16$ square feet. Since there are three squares, the total area from the squares is 3(16) = 48 square feet.

Area of hexagon 3: We divided the hexagon into six equilateral triangles in which each side is 4 feet. Using the 30-60-90 rule, the height of one triangle is $2\sqrt{3}$. So, $A = \frac{1}{2}bh = \frac{1}{2}(2\sqrt{3})(4) = 4\sqrt{3}$ square feet for one triangle. With six triangles, the total area of the hexagon is $6 * 4\sqrt{3} = 24\sqrt{3} = 41.57$ square feet.

Area of semicircles 4: Since we have two semicircles with the same diameter, we will consider them as one complete circle with a diameter of 4 feet. Area of the circle is πr^2. Since the diameter is 4 feet, the radius is 2 feet. Hence, $A = \pi r^2 = \pi(2^2) = 4\pi = 12.56$ square feet.

The total area of the safe zone is the sum of the regions which is 36 + 48 + 41.57 + 12.56 = **138.13 square feet**.

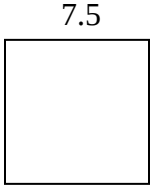
Area of the large circular region is πr^2. Since the diameter is 30 feet, the radius is 15 feet. Hence, $A = \pi r^2 = \pi(15^2) = 225\pi =$ **706.5 square feet**.

So, the theoretical probability is the area of safe zone divided by the area of circle which is $\frac{138.13}{706.5} = .1955 = 20\%$

C. Shaded area = 16 + 16 + 2π +2π = 44.5664 square feet. The shaded area consists of two squares with sides of four so I used the formula $A=s^2$ to determine that they were each 16 square feet. There are also two semicircles with radii of 2 so I used the formula A = $\frac{\pi r^2}{2} = \frac{\pi(4)}{2} = 2\pi$. Then, I added up the areas. The probability of getting the bonus is the probability of getting in shaded region divided by the probability of landing in the safe zone. So,
$\frac{44.5664}{706.8583} = 0.0630 = 6.3\%$

D. The probability for the new area divided by the probability of landing in the safe zone needs to be between 7 and 8 percent. So, 7% of the circle area is .07(706.8583) = 49.4801 square feet and 8% of the circle area is .08(706.8583) = 56.5487 square feet.

Answers may vary, but one example of a new area is shown below.



The probability of this area is (7.5)(7.5) = 56.25 and the probability of landing in this new safe zone is

$\frac{56.25}{706.8583} = 0.0796 = 8\%$

Specific Rubric:

5. Advanced Understanding:

- Correct answer: part A/experimental is 18%, part B/theoretical is 20%, part C/ bonus is 6%, part D/ answer varies for the diagram, procedure, and reason for their choice to achieve the answer.
- All work is shown/ labeling the figure and parts of the figure used for calculation. Computation of the safe zone, unsafe zone, experiential, theoretical, bonus, and desired reduction of the probability.
- Explanation includes how and why the steps were performed.

4. Satisfactory Understanding:

- Correct answer as in score 5 above
- SOME explanation is shown but not complete.

3. Almost Satisfactory Understanding:

- Correct answer as in score 5 above
- ALL work is shown and NO explanation

- Correct answer
- SOME explanation is shown and SOME work

- Incorrect answer due to one calculation error
- SOME explanation is shown and SOME work

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2. Partial Understanding:

- Correct answer
- SOME work is shown and NO explanation

- Correct answer
- NO work is shown and SOME explanation

- Incorrect answer due to two or more calculation errors
- Basic reasoning acceptable but not clear

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1. Minimal Understanding:

- Correct answer
- NO work is shown and NO explanation

- Incorrect answer due to numerous miscalculation errors
- SOME work is shown and SOME explanation

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0. No Understanding:

- Incorrect answer in which the students attempts the task incorrectly with an incorrect explanation or no information. Nothing is correct.
- Blank or off-task responses.