

Open-Ended Template

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Grade Level: 6-8

Content Area: Geometry/Trigonometry

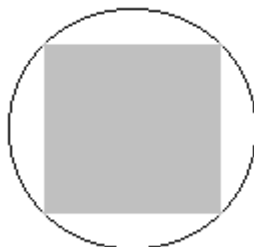
PA Standard(s) addressed: 2.10.8 A and B

NCTM Standard(s) addressed: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Use visualization, spatial reasoning, and geometric modeling to solve problems.

Problem Name: Psycho-delic Carpet Co. Problem

Problem: The P-C Co. is designing a school-spirit carpet. The carpet will be circular in shape with a different colored square in the middle, as shown in diagram 1. As the designer, you will need to determine the cost of the dye for manufacturing the carpet.

Diagram 1:



Directions: For full credit, you *must* do the following:

1. Show or describe each step of your work, even if you did it in your head or used a calculator AND
2. Write an explanation stating the mathematical reason(s) *why* you chose each of your steps.

(Round all answers to the nearest hundredths. Use the Pi symbol on your calculator.)

Part A: The length of one side of the shaded square is 6 feet. Determine the area of the portion of the carpet that is *not shaded*.

Part B: The cost to dye the shaded portion of the carpet is \$1.75 per square yard, and \$1.25 per square yard for the portion *not shaded*. What is the total cost for dyeing the entire carpet?

Problem Solution(s):

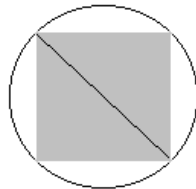
Part A: Work

1. Area of square:

$$A = LW$$

$$A = 6\text{ft} \times 6\text{ft}$$

$$A = 36 \text{ sq ft}$$



2. Area of circle

$$a^2 + b^2 = c^2$$

$$6^2 + 6^2 = c^2$$

$$36 + 36 = c^2$$

$$72 = c^2$$

$$\sqrt{72} = c$$

$$8.49\text{ft} = c$$

$$r = \frac{1}{2} d$$

$$r = \frac{1}{2} (8.49)$$

$$r = 4.25\text{ft}$$

$$A = \pi r^2$$

$$A = \pi 4.25^2$$

$$A = \pi 18.06$$

$$A = 56.74 \text{ sq ft}$$

3. Area of part that is not shaded:

$$56.74 \text{ sq ft} - 36 \text{ sq ft} = 20.74 \text{ sq ft}$$

Explanation:

1. To find the area of the square (shaded portion), I multiplied the length and width. I used the formula for finding area of a square, $A = LW$. Using the definition of a square, I knew that both the length and width were equal to 6 ft.

2. To find the area of the circle, I had to find the radius. I drew a diagonal in my square from the top left corner to the bottom right corner to create a right triangle. This allowed me to use the Pythagorean Theorem to find the length of the hypotenuse, which is also the diameter of the circle.

Once I determined the diameter, I found the radius by taking $\frac{1}{2}$ of the diameter

To figure out the area of the circle, I multiplied π times the radius squared.

3. To determine the area of carpet that is not shaded, I found the difference between the shaded area and the circle.

Part B: Finding the cost of dying the carpet two shades.

Work:

1. Conversion of Units:

Not shaded

$$20.74 \text{ sq ft} / 9 = 2.30 \text{ sq yd}$$

Shaded

$$56.74 \text{ sq ft} / 9 = 6.30 \text{ sq yd}$$

2. Cost of shaded area

$$6.30 \text{ sq yd} \times \$1.75 = \$11.03$$

Cost of area not shaded

$$2.30 \text{ sq yd} \times \$1.25 = \$2.88$$

3. Total Costs to Dye Carpet:

$$\$11.03 + \$2.88 = \$13.91$$

Explanation:

1. Since my price is per square yard and my answer is in square feet, I need to convert the square feet into square yards. I determined there are 9 square feet in a square yard, because a square yard is three feet x three feet. Therefore to find the area of the carpet sections in square yards, I divided my square feet by 9.
2. To find the cost of each section, I multiplied the area of that section by the corresponding price.
3. To figure out the total cost to dye the carpet, I added the cost of each section.

5. Advanced Understanding: Correct answer with all correct procedures/ calculations shown OR described AND a correct explanation that tells "why" each step was taken.

4. Satisfactory Understanding:

- A. Correct answers with all correct procedures/calculations shown OR described AND insufficient or no explanation
- B. Correct answers with most correct procedures/calculations shown OR described AND some explanation

3. Almost Satisfactory Understanding:

- A. Correct answers with most correct procedures/calculations shown OR described AND no explanation
- B. Correct answers with few correct procedures/calculations shown OR described AND some explanation
- C. Incorrect answers with all correct procedures shown OR described AND some explanations BUT with one calculation or copying errors carried through (i.e. student uses diameter instead of radius)

2. Partial Understanding:

- A. Correct answers with few correct procedures/calculations shown OR described AND some or no explanation
- B. Incorrect answer with half or more correct procedures/calculations shown OR described AND some or no explanation (i.e. do not use Pythagorean Theorem to find the length of the diameter)
- C. Incorrect answers with all correct procedures shown OR described AND no explanations BUT with no more than two calculation or copying errors carried through

1. Minimal Understanding:

- A. Correct answers BUT with procedures, calculations or explanations that are not legible or not understandable or missing or incorrect
- B. Incorrect answer with correct procedure to find total costs, BUT incorrect procedure to find cost per square yard with no more than two calculation or copying errors. Some or no explanations
- C. Incorrect answer with incorrect procedure to find total costs, BUT correct procedure to find cost per square yard with no more than two calculation or copying errors. Some or no explanations

- D. Incorrect answer BUT with some information for solving task such as a correct partial procedure (i.e. Pythagorean Theorem is used to find the length of the hypotenuse, but do not use that length further in the problem)
- E. Incorrect answers with correct procedures shown OR described, BUT with three calculation or copying errors. Some or no explanations

0. No Understanding:

- A. Incorrect answer with no correct procedures, calculations, or explanations shown or described. (i.e. Using perimeter rather than area)
- B. Question marks (????), "I don't know," a written "Absent" on the response page (student has read or seen the item), inappropriate, or no response.