

Open-Ended Problem Solving

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Grade Level: 8

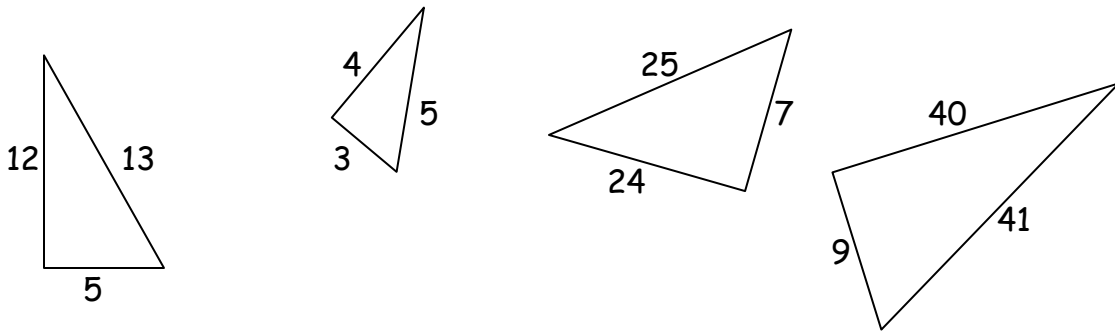
Content Area: Algebra

PA Standard(s) addressed: 2.8 Algebra and Functions

NCTM Standard(s) addressed: Understanding patterns, relations, and functions.

Problem Name: GET A LEG UP ON THE HYPOTENUSE!

Problem: The following table demonstrates examples of special Pythagorean right triangle triplets. If you know two of the three sides of a right triangle, you can always find the third side by using the Pythagorean Theorem. Now, given only the shorter leg, find the two missing sides based on the provided examples.



Shorter Leg	Longer Leg	Hypotenuse
3	4	5
5	12	13
7	24	25
9	40	41
11	<input style="width: 30px; height: 20px;" type="text"/>	<input style="width: 30px; height: 20px;" type="text"/>

Directions: To get full credit you must show or describe each step of your work, even if you did it in your head ("mental math") or used a calculator. You must write an explanation stating the mathematical reason(s) why you chose each of the steps. Write the correct answer in the boxes above.

Problem Solution(s):

Shorter Leg	Longer Leg	Hypotenuse
3	4	5
5	12	13
7	24	25
9	40	41
11	60	61

WORK	EXPLANATION
$12-4=\underline{8}$ $24-12=\underline{12}$ $40-24=\underline{16}$	1. In order to find the missing dimensions of the triangle, I looked for patterns in the numbers on the chart. I noticed a difference of 8 between 12 and 4; a difference of 12 between 24 and 12 and a difference of 16 between 40 and 24.
$40+20=\underline{60}$	2. After examining the differences I realized that the longer leg was increasing by multiples of 4. The next multiple of 4 would be 20. Therefore I added 20 to 40 in order to get the longer leg of the next triangle.
$4+1=\underline{5}$ $12+1=\underline{13}$ $24+1=\underline{25}$ $40+1=\underline{41}$	3. Next, I noticed that the hypotenuse of each successive triangle is one more than the longer leg. Therefore, I added 1 to 60 to get the hypotenuse of 61.
$60+1=\underline{61}$	

Alternate Solutions:

- Steps 1 and 2 but in step three the student solved for the hypotenuse by using the Pythagorean Theorem which is $a^2 + b^2 = c^2$.
- Steps 1 and 2 but student notices that the hypotenuse of each triangle also increases by multiples of four and uses this pattern to find 61.
- Student would first find the hypotenuse by noticing the increase of the multiples of four and subtracting ONE (1) from it to find the longer leg of the triangle.
- Finding the closed form formula:
 Shorter leg = $2n+1$
 Longer leg = $2n^2+2n = 2n(n+1)$
 Hypotenuse = $2n^2+2n+1 = 2n(n+1)+1$

Specific Rubric:

5 -Advanced Understanding, Excellent

Correct answer of 60, 61 with correct procedures [vertical column patterns; OR pattern AND Pythagorean Theorem; OR horizontal column patterns; OR closed form formula; OR trial and error] and calculations shown or described and an explanation, which shows understanding.

4-Satisfactory Understanding

- A. Correct answer with correct procedures and calculations shown or described with some explanation.
- B. Correct answer with partial procedures/calculations shown or described and complete explanations.

3- Almost Satisfactory Understanding

- A. Correct answer with most or all of the work shown or described and no explanation.
- B. Correct answer with a correct procedure and calculations shown or described and some explanation.
- C. Incorrect answer with a correct procedure shown or described and some explanation BUT with one calculation and copying error carried through.

2-Partial Understanding

- A. Correct answer with partial work and calculations shown and no explanation.
- B. Incorrect answer with partial work and calculation and some explanation.
- C. Incorrect answer with a correct procedure and two calculation errors and no explanation.

1-Minimal Understanding

- A. Correct answer with procedures, calculations and explanations that are not legible or not understandable or missing or incorrect.
- B. Incorrect answer, with a correct and critical procedure shown or described with some or no explanation. If there is a calculation with that procedure it must be correct.

0-No Understanding

Incorrect answer with no correct procedures, calculations or explanations shown or described. Also, may include a question mark or "I don't know" written on page.